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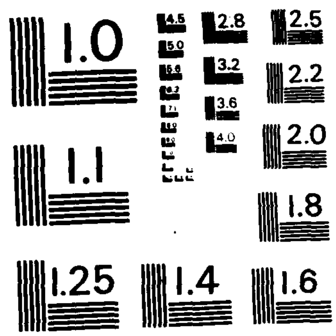

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AD A139491

THE EFFECT OF THE
POISSON APPROXIMATION ON THE
WARTIME ASSESSMENT AND REQUIREMENTS SYSTEM
ASSESSMENT RESULTS

THESIS

AFIT/GST/OS/83M-7

Steven W. Weiss
Capt USAF

Neil A. Youngman
Capt USAF

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THESIS

Presented to the Faculty of the School of Engineering
of the Air Force Institute of Technology
Air University
in Partial Fullfillment of the
Requirements for the Degree of
Master of Science

by

Steven W. Weiss, B.S.
Capt USAF

and

Neil A. Youngman, B.S.
Capt USAF

Graduate Strategic and Tactical Sciences
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Preface

The purpose of this study was to investigate the effect of using a Poisson approximation in the WARS ASSESS subroutine. This approximation is a small part of the ASSESS subroutine which in turn is a small part of the WARS model. Our aim was not to evaluate the preformance of WARS or ASSESS but only to study the effect of this particular approximation. We hope that in some small way we have added to the understanding of this important model.

We would like to thank our advisors, Maj Joseph W. Coleman and Lt Col James N. Bexfield of the Air Force Institute of Technology at Wright-Patterson AFB, Ohio, who have guided this study to completion. Sincere gratitude is also expressed to Maj Douglas Rippy and Capt Steven Schroeder for their expert council. Finally we would like to acknowledge the patience of our wives and families in this demanding undertaking.

Steven W. Weiss

Neil A. Youngman

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Abstract

The purpose of this study was to determine the relative effect of using the Poisson approximation in the Wartime Assessment and Requirements System (WARS) assessment subroutine and to gain a better understanding of the exact error-causing mechanisms involved.

To accomplish this two Fortran computer programs were developed, one to compute the expected number of not mission capable aircraft using the accurate mathematical calculations, the other to compute the same figure using the Poisson approximation. These programs were used to evaluate the approximation caused error for different parts hierarchies and data sets.

The analysis identified two distinct causes for the error induced by the approximation. These sources of error were confirmed by running test cases specifically tailored to eliminate the error-causing characteristics and noting that no approximation error resulted.

The approximation error was found to fluctuate in sign and magnitude for different cases. Sensitivity analysis was performed to identify the sensitive parameters.

THE EFFECT OF THE POISSON APPROXIMATION ON THE WARTIME ASSESSMENT AND REQUIREMENTS SYSTEM ASSESSMENT RESULTS

I Introduction

Background

The War Reserve Materiel Concept. In peacetime an operational unit receives its required spare parts through normal supply channels; however, during contingencies the unit may be deployed to another location and be unable to maintain its established supply lines. If the mobilization is in response to a contingency plan, logistical support is provided through the War Reserve Materiel (WRM) program, which is designed to support the wartime mission until production and airlift can ensure adequate resupply. This initial resupply is accomplished by stockpiling war materiel and prepositioning a portion of it in the theatre of intended use prior to the beginning of hostilities (Ref 1: Ch 1, 1).

WRM is divided into three separate areas, War Readiness Spares Kits (WRSK), Base Level Self-sufficiency Spares (BLSS), and Other War Reserve Materiel (OWRM). WRSK is a special category of WRM which is defined as:

> An air transportable package of WRM spares, repair parts and related maintenance supplies required to support planned wartime or contingency operations of a weapon or support system for a specified period of time pending resupply (Ref 2: Ch 14, 10A).

Current Air Force directives normally specify a 30-day time period without resupply. WRSKs are authorized for specific units which have weapon, support, or mobile command and control systems, and are normally prepositioned with the using unit (Ref 3: Ch 2, 2). Although WRM consists of various types of materiel such as petroleum, munitions, rations, spare parts, and communication equipment, WRSKs contain only mission essential replacement parts (Ref 2: Ch 14, 10A). BLSS is similar to WRSK in that its purpose is also to insure an adequate supply of spare parts for units during wartime operations, however, unlike WRSK, "BLSS is a non-mobile spares package that augments inplace operating spares" (Ref 3: Ch 2, 1) at selected forward operating bases. OWRM is WRM that is not included in WRSK or BLSS.

<u>Current Requirements Methodology</u>. The basic WRM concept is easily understood; however, the actual determination of "which parts" and "how many" to include in a WRM package is a difficult task. The large number of variables and the uncertainties associated with them add to the complexity of the problem. Determining optimal quantities is extremely important because in time of war, shortages of essential spare parts can severely degrade mission accomplishment. Conversely, stockpiling excess quantities or unnecessary

parts is very expensive, especially considering the fact that the current WRSK inventory alone is valued at near $3.6 billion (Ref 4). One can easily see that minor improvements in methodology can result in major improvements in mission capability and substantial dollar savings.

WRSK, BLSS, and OWRM quantities are computed by similar methods, so for the sake of simplicity the subsequent discussion will be limited to WRSK.

Currently the kits are built using a "WRSK candidate list." This is a unique listing for each aircraft type or other system, which has been developed over the years by users and system managers to specify which spare parts are mission essential and are therefore authorized for WRSK. This compilation is then used as a "shopping list" of possible items for inclusion in the kit.

To answer the "how many of each item" question, the Air Force Logistics Command has developed several different analytical techniques. Two techniques currently used are referred to as the conventional method and the marginal analysis method and are described below.

A WRSK built by the conventional method (called a conventional kit) basically includes quantities equal to the maximum expected shortage for each spare part for the entire WRSK period. These quantities are determined in the follow-

ing manner. Using historical data, a mean failure rate per flying hour is calculated for each item by dividing the total number of failures in a certain time period by the number of flying hours in the period. This mean failure rate is multiplied by the number of items per aircraft and the expected wartime Daily Flying Hour Program (DFHP) to arrive at an expected number of failures for each day. If no maintenance capability exists or if the ite is a non-reparable type, the maximum expected shortage for he period is simply the sum of the daily expected failures owever, if maintenance capability does exist, the number of reparable items repaired on each day is computed using an average repair rate (called Base Repair Cycle or BRC) and an initial setup time for a maintenance shop. Knowing the expected number of failures for each day and the expected number of repaired items each day, the maximum expected shortage can be found. This maximum expected shortage is the quantity of that particular item included in the conventional kit. The process is then repeated for each item on the WRSK candidate list for the weapon system under consideration (Ref 5: Ch 2, 1-2; 6: 1-8).

The Marginal Analysis (MA) method is designed to create a new WRSK, with the "same performance" as the conventional WRSK, at a cheaper price. Performance is defined in terms of two parameters, maximum expected WRSK shortage and maximum expected number of Not Mission Capable (NMC) aircraft,

both taken over the desired number of days. For example, if the URSK period to be supported is 15 days and a certian kit is evaluated as indicated in the following table, then the maximum expected number of NMC aircraft is 3.1 and the maximum expected part shortage is 4.2. These two figures represent the performance of this particular kit.

EVALUATION OF WRSK X

DAY	EXPECTED NO. OF PARTS SHORT	EXPECTED NO. OF A/C NMC
1	0	0
2	0	0
3	0	.7
4	1.1	1
5	2	1.3
6	2.3	1.5
7	3	2.1
8	3.2	2.4
9	4	2.9
10	4.2	3.1
11	3.8	2.3
12	3.3	2.1
13	3	1.9
14	2.4	1.5
15	2	1.4

Table 1-1

Note: The decrease after day 10 could be caused by repair capability becoming operational or a decrease in the DFHP.

We will use aircraft for illustrative purposes although WRSK is not specifically limited to aircraft and flying units.

An MA WRSK is determined by first building a conven-

tional kit and evaluating it with respect to the shortage and NMC parameters, that is, determining the maximum expected number of broken aircraft and the maximum expected number of parts short, over the period for this kit. These parameters are then used as lower bounds for the MA kit, which is built from a small initial base, normally adding items one at a time. The next item to be added is the one that provides the greatest reduction in the parameters per dollar spent. The reduction in expected shortage due to the addition of a given part to a WRSK is found by assuming that failures occur according to a Poisson probability distribution and using expected value calculations. The reduction in expected NMC (E[NMC]) aircraft is found in the same manner after making the assumption of full cannibalization of parts. This assumption concentrates all the shortages or failures into the minimum number of aircraft and hence simplifies the problem. Parts which cannot be cannibalized are not considered in MA, their quantities determined solely by the basic conventional method. This process of adding the most cost efficient item to the kit is continued until the kit has the same performance as the conventional kit but at less cost (Ref 5: Ch 3, 1; 6).

The current methodology for determining WRSK requirements has three inherent problem areas. The first shortcoming is the treatment of cannibalization. In the present methodology full cannibalization is assumed and any part

which is considered "non-cannable" is not included in the optimization process. The WRSK quantities for these non-cannable parts are determined by the conventional method. This procedure results in the non-cannable portion of a WRSK being sub-optimal and therefore more costly than needed (Ref 7).

A second problem is the relationship between Line Replaceable Units (LRUs) and Shop Replaceable Units (SRUs). An SRU is a component part of an LRU but because of its inaccessibility it can be repaired only in the maintenance shop. For example, the LRU is the main assembly or black box which can be replaced on the flight line while the SRU is the subassembly or component in the black box which can be replaced only in the shop. Currently SRUs and LRUs are considered as unrelated parts whereas they should have some sort of indenture relationship. An SRU should not fail without its corresponding LRU also failing. For example, if an amplifier in a radio fails, the entire radio should also fail. The present methodology does not take this into account. The fact that LRUs are themselves a source of parts for SRUs is also not addressed (Ref 7).

The final problem area is inclusion of a goal which limits the number of backordered parts (called stock due out or SDO). The MA method builds a new WRSK with the same level of performance as the conventional kit but at a

cheaper price (Ref 5). The rationale behind this method is that the same number of aircraft will be NMC because of part shortages, but the parts short will be expensive parts rather than cheap parts. Performance is defined in terms of two parameters, maximum expected SDO and maximum expected number of NMC aircraft, both taken over the WRSK period under consideration. The requirement that the MA kit not exceed the conventional kit SDO goal is inappropriate because an SDO goal is not an accurate measure of a WRSK's ability to support the mission. Imposing a maximum SDO level unduly restricts the WRSK and costs extra dollars (Ref 7).

The Air Force realized the shortcomings in the present methodology and began development of the Wartime Assessment and Requirements System (WARS) to improve WRSK and overall WRM performance.

Wartime Assessment and Requirements System (Ref 8). WARS is a mathematical optimization model used to determine the quantities of WRM items in a kit which will yield the best performance per dollar spent. The Air Force Logistics Command is currently developing this model which will use a MA method of optimization similar to the method previously described. The main differences are that WARS uses an NMC goal, whereas the present method uses both an NMC goal and an SDO goal, and that WARS can accomodate non-cannable and

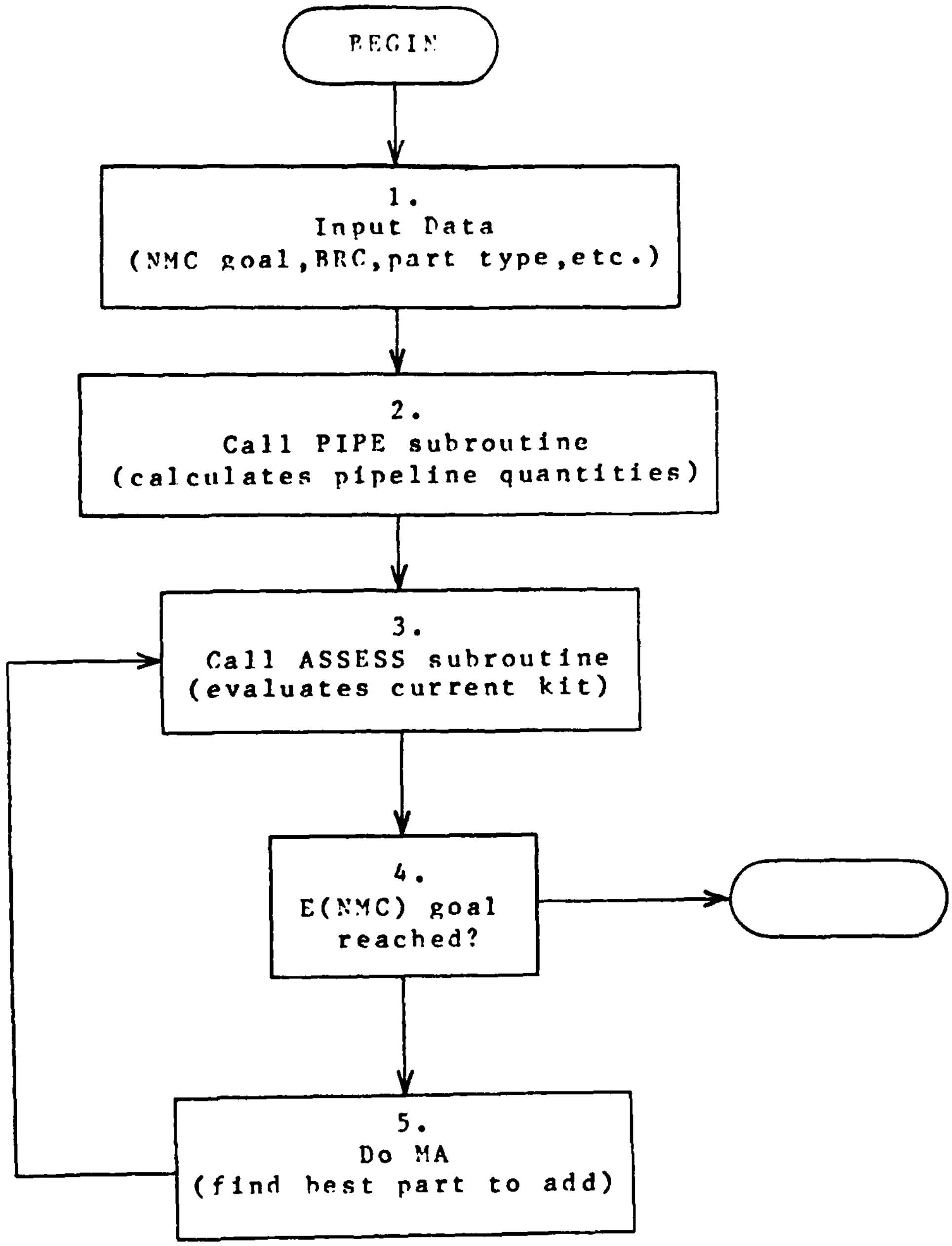
indentured items.

The basic structure of the WARS model is depicted in Figure 1-1. For simplicity the discussion will be limited to aircraft spare parts, although WARS is applicable to all types of WRM. The required input data in block 1 includes:

<u>General</u>
Period covered (days)
No. of assigned aircraft
Daily Flying Hour Program (DFHP)
Maximum expected NMC goal

<u>Individual Item Data</u>
National Stock Number (NSN)
Parent item NSN
Hierachy position on aircraft
- Type I-an SRU assigned to an LRU
- Type II-an LRU assigned to a module
- Type III-an LRU or module assigned to an engine
- Type IV-an LRU or engine assigned to an aircraft

Kind of item
- LRU, SRU, engine, module
- reparable,irreparable
- cannable,non-cannable
- quantity per higher assembly (QPHA)
- quantity per aircraft (QPA)
- unit cost
- Base Repair Cycle (BRC)
- Depot Repair Cycle (DRC)
- transportation time to depot
- total demand per hundred flying hours
- Base Repair Rate (BRR)
- Base Condemnation Rate (BCR)
- Depot Condemnation Rate (DCR) (Ref 9: 8)



Basic Structure of WARS

Figure 1-1.

This input data is used by the "PIPE" subroutine in

block 2 to calculate the maximum expected number of each item "that have been removed from the aircraft due to failures and have not yet been returned to a serviceable condition available for replacement onto the aircraft." This quantity is called the "pipeline quantity" or "MU" and is the expected value on which the probability distribution of demands is based. (Ref 9: 10) For example, if the support period under consideration is ten days and the kit contains two parts, the PIPE subroutine might calculate the following data:

EXPECTED NO. OF PARTS UNAVAILABLE

DAY	PART A	PART B
1	0	.1
2	.3	.4
3	.7	1.3
4	1.1	2.7
5	.8	3.1
6	.7	2.6
7	.7	2.2
8	.6	2.2
9	.6	2.1
10	.5	2.1

Table 1-2

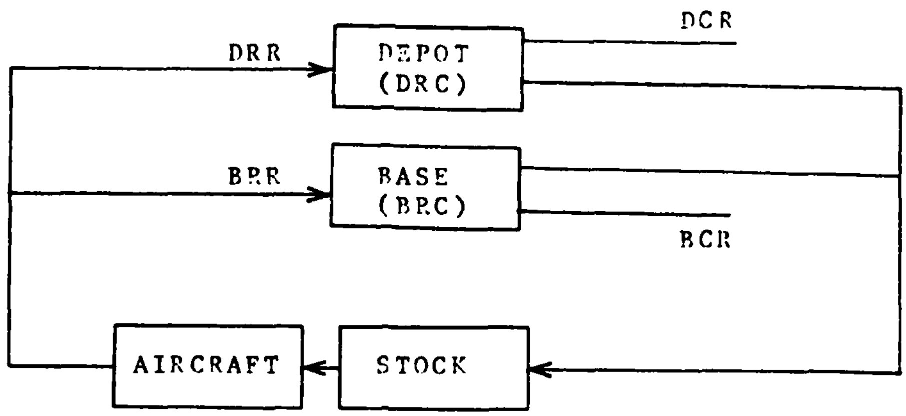
Note: the decrease around day four could again be caused by the arrival of repair capability or a decrease in the DFHP.

The maximum expected number of Part A unavailable is 1.1 which occurs on day four, while the maximum expected numberr of Part B unavailable is 3.1 which occurs on day five. So the pipeline quantities or MUs for Parts A and B are 1.1 and 3.1 respectively.

Block 3 uses the MU values calculated in PIPE to determine the expected number of NMC aircraft using the "ASSESS" assessment subroutine, based on the present stock in the kit. (Ref 9: 11) The PIPE and ASSESS subroutines will be explained in greater detail later.

Block 4 checks to see if the E[NMC] goal set by the operator is met. If so, the kit stock levels and the E[NMC] figure are output, otherwise the MA process is performed to determine which item should be added to the kit to obtain the greatest decrease in E[NMC] per dollar spent (block 5). Once this is accomplished, the kit is re-evaluated by ASSESS (block 3) and the process is repeated.

PIPE calculates the daily expected number of each item removed from aircraft due to failure and not yet serviceable. MU is then set equal to the maximum of the daily quantities for the desired period (Ref 9). Figure 1-2 depicts the flow of failed parts through the "pipeline."



Parts Pipeline

Figure 1-2.

Suppose a certain part fails on an aircraft. It is removed and enters the pipeline. Usually the part will be repaired at base level, however a certain percentage will require depot level maintenance. The percentages that go to the base and to the depot are called the Base Repair Rate (BRR) and the Depot Repair Rate (DRR) respectively. At both the base and the depot a certain percentage will be condemned. These percentages are known as the Base Condemnation Rate (BCR) and the Depot Condemnation Rate (DCR) respectively. The time required to repair a particular item at base level is called the Base Repair Cycle (BRC) and similarly a Depot Repair Cycle (DRC) at the depot level. Based on the DFHP and the above input parameters, PIPE calculates MU for each item.

The cornerstone of the MA process of optimization is the ability to calculate the E[NMC] for any stock level in a kit. The WARS model performs this task using the subroutine ASSESS. Once the maximum pipeline quantities are determined for each part, this information is used by the ASSESS subroutine to calculate the maximum E[NMC] for a given stock level using the following expected value formula:

$$E[NMC]=0xP[0 \text{ NMC a/c}] + 1xP[1 \text{ NMC a/c}] + 2xP[2 \text{ NMC a/c}] + 3xP[3 \text{ NMC a/c}] + \ldots + UExP[UE \text{ NMC a/c}] \quad (1)$$

where UE is the number of aircraft assigned to the unit (Ref 9: 16). Computing the above probabilities is extremely complex because of the possibility of some parts being cannibalized from other aircraft and the indenture relationship between different parts.

Figure 1-3 shows some indenture relationships in a simple component hierarchy for a fictitious aircraft. Note that SRU 5 and SRU 6 are component parts of LRU 5 which is itself a component part of module 1. Also note that the type of item is given on the line above the item. For example, SRU 5 and SRU 6 are type I items while LRU 5 is a type II item. Recall that type designations merely reflect the indenture level of the part.

When discussing indenture relationships it is important

to understand that if an SRU such as SRU 3 has failed and no replacement is available, its parent parts up the tree (LRU 3 and engine 1) also fail. The converse is not true. If a parent part such as engine 1 fails, it has not necessarily failed because of a failure of one of its component parts. There are numerous pieces in an engine assembly itself besides the lower indentured items which could fail. Said another way, all the parts that make up a parent item are not identified as separate components and shown on a parts hierarchy.

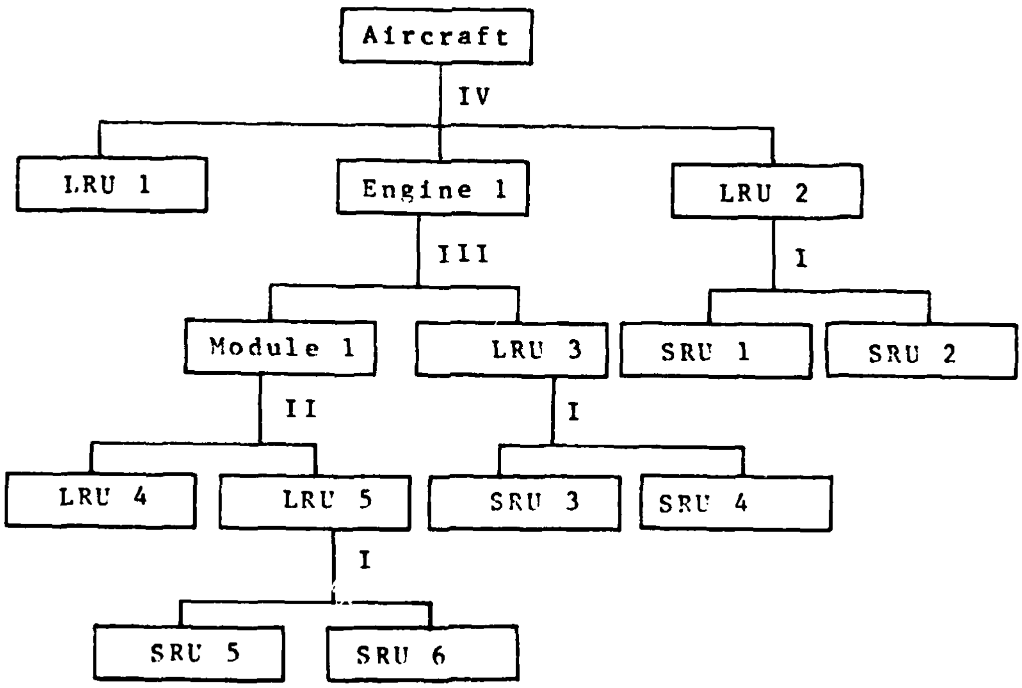


Figure 1-3

Hierarchial Representation of an Aircraft

Before discussing the computational methodology some additional terms need to be defined (Ref 10):

higher assembly (HA)-refers to an item that contains parts that can only be reached by taking this item apart, sometimes called a parent with respect to immediately lower parts.

quantity per higher assembly (QPHA)-the quantity of a given part contained within each of its parent parts.

quantity per aircraft (QPA)-the quantity of a given part contained on one aircraft.

backorder-an unfilled demand or shortage at base level.

perfectly cannibalizable part (PCP)-a part that can be cannibalized from one HA to another and which has the same QPHA on all HAs containing the item.

non-cannibalizable part (NCP)- a part that can not be removed in a serviceable condition from one aircraft or HA to prevent a backorder on another aircraft. While it may be physically possible to cannibalize such an item, the item may be designated as a NCP because the item would be damaged when it is removed or the time to cannibalize it is considered excessive.

imperfectly cannibalizable part (MCP)-a part that can be cannibalized from one HA to another and which does not have the same QPHA on all HAs containing the item. Also called myopically cannibalizable part.

national stock number (NSN)-a unique number by which each part procured by the federal government is identified.

unique item-an NSN that appears in only one position in the aircraft parts hierarchy.

non-unique item-an NSN that is used in more than one hierarchial position on an aircraft.

A comprehensive list of terms is given in the glossary in

appendix A.

In describing the assessment procedure the steps are presented in the reverse of the order of occurrence. This is done to continuously provide the reader with the rationale for each successive step. A detailed account of the variables and formulas used in the assessment routine is included in appendix B. This general description will start with the final answer and work backward through lower and lower indenture levels as additional information is needed. In the actual routine the lowest level is evaluated first and the information obtained from it used in each successive level up the tree until the required information is available at the top level to compute E[NMC]. The procedure will be explained for the top indenture level but the other levels are evaluated in a similar manner (Ref 9,11).

E[NMC] for a given stock level is found using the following steps:

1. If UE is the number of aircraft in the unit under consideration, then using the expected value formula for discrete random variables (Ref 12: 96):

$$E[NMC]=0\times P[0\ NMC\ a/c] + 1\times P[1\ NMC\ a/c] + 2\times P[2\ NMC\ a/c] + 3\times P[3\ NMC\ a/c] + \ldots + UE\times P[UE\ NMC\ a/c] \quad (1)$$

2. The probabilities that a certain number of aircraft are NMC are merely the probability function (PF) of the number

of aircraft NMC and can easily be calculated if the cumulative distribution function (CDF) is known (Ref 13: 60). Therefore if the CDF is known the PF can be calculated and the individual probabilities used in the right side of equation 1.

3. The CDF needed in step 2 is obtained by computing:

$$P(\#\ a/c\ NMC<x) = P(\#\ a/c\ NMC\ for\ NCP\ or\ MCP<x) \times P(\#\ a/c\ NMC\ for\ PCP<x) \qquad (2)$$

For all $x=0,1,2,\ldots,UE$

The number of aircraft NMC is the maximum of the number of aircraft NMC for PCP and the number of aircraft NMC for NCP or MCP. So symbolically:

$$P(Max[Y,Z]<c) = P(Y<c \text{ and } Z<c) = P(Y<c) \times P(Z<c)$$

In effect this is multiplying two independent CDFs by elements to arrive at a resultant CDF. The CDFs are combined in this manner because the distribution of the maximum of the two random variables (max[Y,Z]) is required and not the sum (Y+Z) (Ref 12: 253, 14: 310). This is true because cannibalization can be used to consolidate the failures onto fewer airframes. Consider an example combining the following two CDFs:

CDF 1

x	0	1	2
P(X<x)	.1	.5	1.0

CDF 2

x	0	1	2
P(X<x)	.3	.7	1.0

Multiplying elements yields the following:

RESULTANT CDF

x	0	1	2
P(X<x)	.03	.35	1.0

Converting this CDF to a PF:

RESULTANT PF

x	0	1	2
P(X=x)	.03	.32	.65

The same answer could be obtained by using the following PFs,

PF 1

x	0	1	2
P(X=x)	.1	.4	.5

PF 2

x	0	1	2
P(X=x)	.3	.4	.3

and performing the following calculations:

$$P(X=0) = .1 \times .3 = .03$$

$$P(X=1) = .1 \times .4 + .4 \times .3 + .4 \times .4 = .32$$

$$P(X=2) = .1 \times .3 + .4 \times .3 + .5 \times .3 + .5 \times .3 + .5 \times .4 = .65$$

These results agree with the resultant PF above as expected.

The CDFs are independent rather than mutually exclusive because an aircraft could be broken for several different types of parts at the same time. For example, an aircraft could initially be broken for an NCP, and subsequently have a PCP cannibalized from it. Thus the aircraft would be broken for two different types of parts.

4. At this point the procedure splits into two branches. We will first trace through the procedure for determining the last term in equation 2 and return later to explain how the other term on the right hand side of the equation is

obtained. The steps in the first branch will be numbered "5a", "6a", etc. while the second branch will be numbered "5b", "6b", etc.

5a. Tracing the first branch:

$$
\begin{aligned}
P(\#\text{ a/c NMC for PCP} \leq x) = \; & P(\#\text{ a/c NMC for PCP } 1 \leq x) \times \\
& P(\#\text{ a/c NMC for PCP } 2 \leq x) \times \\
& \quad \vdots \\
& P(\#\text{ a/c NMC for PCP last} \leq x) \qquad (3)
\end{aligned}
$$

For all $x=0,1,2,\ldots,UE$

The CDFs on the right side of the equation are again independent, and it is the maximum of the random variables which is of interest using the same logic as in step 3 above.

At this point only the top indenture level parts are considered and only PCPs.

6a. Each of the probability expressions in equation 3 can be calculated by the formula

$$P(\#\text{ a/c NMC for PCP } i \leq x) = \sum_{y=0}^{x} \frac{u^{y} e^{-u}}{y!} \qquad (4)$$

Again for all $x=0,1,2,\ldots UE$

if it is assumed that the number of NMC aircraft for any PCP is Poisson distributed and the mean is known for each part. Since the CDF in equation four is of finite length, WARS adds the probabilities in the far tail of the Poisson distribution into the last interval used. ASSESS currently assumes a Poisson distribution but it will be able to use the Negative Binomial distribution in a later version.

7a. The "u" or TOTMU values for each PCP used in equation 4 are determined by using the formula:

$$TOTMU = AMU + AWP \tag{5}$$

where AMU is the "actual MU" or expected pipeline quantity for the part calculated by the PIPE subroutine and AWP is the number "awaiting parts" or unavailable due to a lack of lower indentured parts. The AWP figures for each part are the only results of the calculations for a lower indenture level used in the next higher level. All items with no subassemblies have an AWP figure of zero. For the top level there is only one AWP figure and is called E[NMC], the number of aircraft awaiting lower indentured parts.

5b. Returning to step 4 and tracing the branch for the other term on the right hand side of equation 2:

$$P(\#\ a/c\ NMC\ for\ NCP\ or\ MCP \leq x) =$$

$$\sum_{n=0}^{x} \binom{UE}{n} \left[P(NMC\ for\ NCP\ or\ MCP)\right]^{n} \left[1-P(NMC\ for\ NCP\ or\ MCP)\right]^{UE-n} \quad (6)$$

A CDF is obtained by using this binomial formula for x=0,1,2,...,UE. For the top indenture level there is only one HA, the aircraft itself, and hence one CDF. For lower levels there will be a CDF for each HA in that level and UE in the equation will be replaced by the quantity of each HA in the fleet.

6b.

$$P(NMC\ for\ NCP\ or\ MCP) = 1-\left[P(all\ NCP\ avail.) \times P(all\ MCP\ avail.)\right] \quad (7)$$

7b.

$$\begin{aligned} P(all\ NCP\ avail.) = {} & P(1st\ NCP\ avail.) \times \\ & P(2nd\ NCP\ avail.) \times \\ & \vdots \\ & P(last\ NCP\ avail.) \end{aligned} \quad (8)$$

and

$$P(\text{all MCP avail.}) = P(\text{1st MCP avail.}) \times P(\text{2nd MCP avail.}) \times \cdots \times P(\text{last MCP avail.}) \qquad (9)$$

8b. In equation 8 if all parts have unique QPHAs:

$$P(\text{ith NCP avail.}) = \left[1 - \frac{\text{expected shortages(i)}}{\text{quantity of part i in fleet}}\right]^{\text{QPHA(i)}} \qquad (10)$$

with

$$\text{expected shortages} = \sum_{x=\text{stock level}}^{\text{stock level + qty on fleet}} (\text{x-stock level})\ P(\text{x demands}) \qquad (11)$$

and P(x demands) determined from the total pipeline quantity in equation 5. The bracketed expression in equation ten will never be negative because the expected shortage can never exceed the quantity in the fleet.

If there are some multiple QPHA parts, equation 10 becomes slightly more complex.

In equation 9:

$$P(\text{ith MCP avail.}) = 1 - \frac{\text{expected \# of HAs missing items}}{\text{total \# of HAs of part i}} \qquad (12)$$

The numerator in equation 12 can be found if the PF is known. The PF can be calculated since the PF of the number of demands is known and the number and QPHA of each HA is known. ASSESS assumes all MCPs are cannibalized first from the HAs with the greatest quantity inside.

The PFs of the number of demands used in equations 11 and 12 all need the TOTMUs from equation 5. Therefore the AWP result, or E[NMC], of the top level depends solely on the AMUs provided by the PIPE subroutine and the AWP results of the next lower level.

The preceding assessment methodology makes the following approximations (Ref 15,16):

1. The probability distributions are reset to Poisson from one indenture level to another as explained below.

2. The backorders of a nonunique item have a Poisson distribution in each hierarchy position with that stock number.

3. The probability distribution of HAs awaiting parts due to shortages of NCP or MCP is binomial.

It is the first assumption which is of interest. For each part i, the WARS assessment routine calculates the PF (actually the CDF) of the number of parts AWP and uses it to compute a mean AWP (frequently called Expected Back Order

(EPO)). Adding the mean of this AWP distribution to the mean of the pipeline quantity for part i in equation 5 results in a combined mean which is used to generate a Poisson distribution for its parent part. This Poisson distribution is used as an approximation of the true distribution. The demands of the subassemblies of part i are Poisson but when the stock level of the subassemblies are taken into account the backorder distribution may not be Poisson. Combining subassembly backorder distributions results in an AWP distribution for part i that may not be Poisson.

The correct way of combining the AWP and the AMU PFs to get a TOTMU PF is as follows:

$$\begin{aligned}
P(TOTMU=0) &= P(AWP=0)xP(AMU=0) \\
P(TOTMU=1) &= P(AWP=0)xP(AMU=1) + P(AWP=1)xP(AMU=0) \\
P(TOTMU=2) &= P(AWP=0)xP(AMU=2) + P(AWP=1)xP(AMU=1) \\
&\quad + P(AWP=2)xP(AMU=0) \qquad (13) \\
&\quad \cdot \\
&\quad \cdot \\
&\quad \cdot
\end{aligned}$$

Objective

This PoissonSapproximation of the number of demands of a parent part, when carried through several indenture levels, may induce a significant error in the final E[NMC] figure and could therefore degrade the ability of ASSESS to evaluate the performance of a kit. Since the optimal kit

building capability of WARS hinges on the ability to accuritely evaluate kit performance, the effect of this approximation is obviously an important area to investigate.

AFLC/XRS has made a cursory check of most of the approximations used in the assessment routine but the check was only made for one relatively small parts hierarchy (Ref 15). The Poisson approximation was always evaluated in conjunction with other approximations and was never evaluated separately.

The objective of this study is not to evaluate how well WARS builds optimal WRM packages nor is it to evaluate how well its assessment routine can determine a kit's performance. Rather, the purpose of this thesis is to investigate the effect of resetting the demand probability distribution of every HA to a Poisson distribution at each indenture level. The approximation's effect on the WARS assessment results will be evaluated for a variety of conditions to determine its sensitive parameters.

Scope

Although several approximations are made in the assessment routine, the Poisson approximation will be the only one investigated. In order to eliminate the effects of the other approximations on the results, this study will be limited to fully cannibalizable, unique parts with unique

QPHAs. If non-cannibalizable parts were allowed, the effect of approximation 3 would also be included in the results. If non-unique parts were allowed, approximation 2 would affect the results. Likewise, if parts with several different QPHAs were allowed, the results would again be affected by approximation 3.

The WARS model builds optimal packages for WRSK, BLSS, and OWRM. Since differences between these kits are incorporated into the part pipeline quantities, our study will encompass all types of WRM.

Approach and Presentation

To evaluate the effect of the approximation two computer models were developed. One model, called APPROX, evaluated a package of parts using the WARS procedure with the Poisson approximation. The other model, called EXACT, evaluated a package of parts using exact probability calculations. Both models output the demand probability distributions at each hierarchial position as well as the final E[NMC] figure.

To determine the effect of the approximation, the same data was input to both models and the differences noted. By constructing different aircraft hierarchies and using different part data, sensitivity to the following characteristics was evaluated:

- depth of indenture of the parts hierarchy
- SA to HA ratio for higher levels
- stock level of each part
- QPHA of each part

All numerical results are included in appendices E through G, and summaries of the results are presented in chapter three in tabular form.

II Description of Computer Programs

In order to quickly and accurately calculate the effect of the Poisson approximation for different pipeline quantities, stock levels, and hierarchial structures, two computer programs were developed. The first program, called APPROX, uses the same computations and Poisson approximation as the WARS ASSESS subroutine. The other program, called EXACT, uses similar computations but calculates the exact probabilities by not using the approximation. Both programs evaluate backorder PFs for each part in the aircraft parts hierarchy assuming all parts are cannibalizable, all parts are unique, and no parts have multiple QPHAs. The two programs are described below.

APPROX Computer Program

APPROX evaluates a certain parts hierarchy one part at a time beginning with the lowest indentured parts, evaluating an entire indenture level before moving up to the next level. It does this by storing all data and probability distributions in a three-dimensional array called "A". Figure 2-1 illustrates the array format used to store each part's information. This figure will be used later to help explain the logical steps of APPROX. The actual APPROX program and a listing of computer variables used in the program are included in appendix C.

The following is a list of definitions of the acronyms used in Figure 2-1:

A - three-dimensional data and probability array.

AMU - A variable denoting the basic mean pipeline quantity for an item, excluding AWP. Also called "actual mu" or MU.

APPROX - program which calculates approximate E[NMC].

AWP - A variable denoting the expected number of items "awaiting parts."

BO - backorder.

CDF - Cumulative Distribution Function

DISTL - desired length of probability distributions in array A.

EBO - expected number of backorder.

EXACT - program which calculates exact E[NMC].

HA - Higher Assembly.

MU - pipeline quantity from which the Poisson distribution is generated. Used interchangeably with AMU.

PF - Probability Function.

PUP - position of a subassembly under a parent item, for example, if a parent item has two subassemblies(SA), the first SA will have a PUP of 1 and the second SA will have a PUP of 2.

QPHA - Quantity Per Higher Assembly. This is the number of each item on its respective higher assembly.

SA - Subassembly.

SL - stock level for a specific part.

TOTMU - A variable denoting the total expected pipeline quantity for an item. It is equal to AMU + AWP.

Type (of an item) - The type of an item refers to its indenture relationship as follows:

4 = An LRU or engine attached directly to the aircraft.

3 = An LRU or module attached to a type 4 item.

2 = An LRU attached to a module.

1 = An SRU attached to any LRU.

Format for Each Part in Array A

Program APPROX

For Part i	0	1	2	3	4	5	6	7	. . .	DISTL
1		Type	#SA	QPHA	MU	SL	HA#	PUP		
2	← SA #1 BO CDF →									
3	← SA #2 BO CDF →									
4	← SA #3 BO CDF →									
5	← SA #4 BO CDF →									
6	← SA #5 BO CDF →									
7	← AWP CDF →									
8	← AWP PF →									
9		AWP	TOTMU							
10	← TOTMU PF →									
11	← TOTMU CDF →									
12	← BO CDF →									
13	← BO PF →									
14	Var	EBO	$\frac{\text{Var}}{\text{Mean}}$							

Figure 2-1

Referring to Figure 2-1, for each part in the hierarchy the following tasks are accomplished in APPROX:

1. Multiply lines 2 through 6 (column by column) and store the results in line 7. This converts the BO CDFs for

all subassemblies of this part into an AWP CDF for this part.

2. The AWP CDF (line 7) is converted to an AWP PF (line 8).

3. The expected number of parts AWP is calculated from the AWP PF (line 8) and stored in line 9 column 1.

4. Th expected number of parts AWP from step 3 is added to the expected pipeline quantity for this part (AMU) from line 1, column 4, and the result (TOTMU) is stored in line 9, column 2.

5. TOTMU from step 4 is used as the parameter to generate a Poisson TOTMU PF which is stored in line 10.

6. The TOTMU PF is converted to a TOTMJ CDF (line 11).

7. The TOTMU CDF from line 11 is adjusted for stock level and QPHA to yield a BO CDF for this part (line 12).

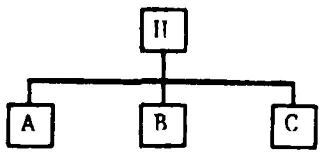
8. The BO CDF (line 12) is converted to a BO PF (line 13).

9. The variance, mean, and variance to mean ratio for the BO PF are calculated and stored in line 14, columns 0 through 2 respectively.

10. The BO CDF (line 12) is transferred to its parent part to be used as a subassembly BO CDF in step 1.

These same 10 steps are done for each part, but they must be done for the type I (most deeply indentured) parts first. After all type I parts have been processed, the type II parts are considered, and so on through the higher part types. The highest part type in WARS is type IV, but for the purposes of the APPROX program the aircraft itself is considered a type V part and is processed last.

To better understand the 10 steps above, an example is presented. Consider the parts hierarchy shown below and its associated parts data:



PARTS DATA

Part	OPHA	MU	Stock
H	1	1	2
A	1	1	0
B	2	2	1
C	2	1	0

The computations start with the type I parts A, B, and C. For part A there are no subassemblies so the subassembly RO CDFs are

x	0	1	2	3	4
P(#BO≤x)	1.000	1.000	1.000	1.000	1.000 . . .

Multiplying them columnwise in step 1, yields an AWP CDF of

x	0	1	2	3	4
P(#AWP≤x)	1.000	1.000	1.000	1.000	1.000 . . .

In step 2 the AWP CDF is converted to the following AWP PF:

x	0	1	2	3	4
P(#AWP≤x)	1.000	0.000	0.000	0.000	0.000 . . .

Step 3, the expected number of parts AWP is calculated to be 0.

Step 4, TOTMU = AMU + AWP = 1 + 0 = 1.

Step 5, using TOTMU = 1, the following Poisson TOTMU PF is generated:

x	0	1	2	3	4
P(#demands=x)	.3679	.3679	.1839	.0613	.0153 . . .

Step 6, the above PF is converted to a TOTMU CDF:

x	0	1	2	3	4
P(#demands≤x)	.3679	.7358	.9197	.9810	.9963 . . .

Step 7, since stock = 0 and QPHA = 1, the BO CDF is identical to the TOTMU CDF:

x	0	1	2	3	4
P(#BO≤x)	.3679	.7358	.9197	.9810	.9963 . . .

Step 8, the BO PF is then:

x	0	1	2	3	4	
P(#BO=x)	.3679	.3579	.1839	,0613	.0153	. . .

Step 9 Variance =1

Mean or EBO = 1

Variance to mean ratio = 1

Step 10, the BO CDF calculated in step 7 will be the distribution used in the calculations for part H. Steps eight and nine only need to be accomplished for the aircraft itself, but are performed for each part for analysis reasons.

For part B, the results of the steps are:

Step 1

x	0	1	2	3	4	
P(#AWP≤x)	1.000	1.000	1.000	1.000	1.000	. . .

Step 2

x	0	1	2	3	4	
P(#AWP=x)	1.000	0.000	0.000	0.000	0.000	. . .

Step 3, AWP = 0.

Step 4, TOTMU = AMU + AWP = 2 + 0 = 2.

Step 5

x	0	1	2	3	4	
P(#demands=x)	.1353	.2707	.2702	.1805	.0902	. . .

Step 6

x	0	1	2	3	4	
P(#demands≤x)	.1353	.4066	.6767	.8571	.9473	. . .

Step 7, since the stock level = 1 and the QPHA = 2, the BO CDF is

x	0	1	2	3	4	
P(#BO≤x)	.4060	.8571	.9834	.9989	.9999	. . .

Step 8

x	0	1	2	3	4	
P(#BO=x)	.4060	.4511	.1263	.0155	.0011	. . .

Step 9 Variance = .5442

Mean or EBO = .7546

Variance to mean ratio = .7211

Step 10, the BO CDF from step 7 will be used in calculations for part H.

For Part C

Steps 1 through 3 again yield the same results as above.

Step 4, TOTMU = AMU + AWP = 1 + 0 = 1.

Step 5

x	0	1	2	3	4	
P(#demands=x)	.3679	.3679	.1839	.0613	.0153	. . .

Step 6

x	0	1	2	3	4	
P(#demands≤x)	.3679	.7358	.9197	.9810	.9963	. . .

Step 7, stock level = 0 and OPHA = 2, so

x	0	1	2	3	4	
P(#BO≤x)	.3679	.9197	.9963	.9999	1.000	. . .

Step 8

x	0	1	2	3	4	
P(#BO=x)	.3679	.5518	.0766	.0036	.0001	. . .

Step 9 Variance = .3790

Mean or EBO = .7162

Variance to mean ratio = .5292

Step 10, again the BO CDF from step 7 will be used in the calculations for part H.

Now, type II parts are considered.

For part H

Step 1, using the subassembly BO CDFs previously calculated:

x	0	1	2	3	4	
P(#A BO≤X)	.3679	.7358	.9199	.9810	.9963	. . .
P(#B BO≤x)	.4060	.8571	.9834	.9989	.9999	. . .
P(#C BO≤x)	.3679	.9897	.9963	.9999	1.000	. . .
P(#H AWP≤x)	.0549	.5800	.9012	.9799	.9963	. . .

Step 2

x	0	1	2	3	4	
P(#H AWP=x)	.0549	.5251	.3212	.0787	.0164	. . .

Step 3, AWP = 1.4885.

Step 4, TOTMU = AMU + AWP = 1.0 + 1.4885 = 2.4885.

Step 5

x	0	1	2	3	4	
P(#demands=x)	.0830	.2066	.2571	.2133	.1327	. . .

Step 6

x	0	1	2	3	4	
P(#demands≤x)	.0830	.2897	.5468	.7600	.8927	. . .

Step 7

x	0	1	2	3	4	
P(#BO≤x)	.5468	.2133	.1327	.0660	.0274	. . .

Step 8

x	0	1	2	3	4	
P(#BO=x)	.5468	.2133	.1327	.0660	.0274	. . .

Step 9 Variance = 1.4466

Mean or EBO = .8612

Variance to mean ratio = 1.6799

In this example there are no type III, IV, or V parts so the EBO of part H is the desired result.

EXACT Computer Program

The EXACT computer program is also included in appendix C. This program is similar to the APPROX program and uses the same computer variables previously listed for APPROX.

Figure 2-2 shows the array format for EXACT. Note that it differs from the APPROX format only on line 9, where the AWP and TOTMU figures are replaced by the AMU PF.

Format for Each Part in Array A

Program EXACT

For Part i	0	1	2	3	4	5	6	7	. . .	DISTL
1		Type	#SA	OPHA	MU	SL	HA#	PUP		
2	⟵ SA #1 BO CDF ⟶									
3	⟵ SA #2 BO CDF ⟶									
4	⟵ SA #3 BO CDF ⟶									
5	⟵ SA #4 BO CDF ⟶									
6	⟵ SA #5 BO CDF ⟶									
7	⟵ AWP CDF ⟶									
8	⟵ AWP PF ⟶									
9	⟵ AMU PF ⟶									
10	⟵ TOTMU PF ⟶									
11	⟵ TOTMU CDF ⟶									
12	⟵ BO CDF ⟶									
13	⟵ BO PF ⟶									
14	Var	EBO	$\frac{\text{Var}}{\text{Mean}}$							

Figure 2-2

The computational steps used in EXACT are similar to those in APPROX except that steps 3 through 5 are replaced by the following two steps:

Step 3.5 - The Poisson AMU PF is generated from the AMU

parameter in line 1, column 4 and stored in line 9.

Step 4.5 - The AMU PF (line 9) and the AWP PF (line 8) are combined as mutually exclusive events to get a resultant TOTMU PF. The probabilities of all possible combinations are added together to get the probability of a certain number of demands on the particular part for any reason (such as missing subassemblies or the part being broken itself). These combinational probabilities are calculated as follows:

P(0 demands) = P(0 AMU) x P(0 AWP)

P(1 demand) = P(1 AMU) x P(0 AWP) + P(0 AMU) x P(1 AWP)

.

.

.

Using the same data and parts hierarchy as in the APPROX example, the calculations were repeated using the EXACT method. Once again, the example starts with type I parts A, B, and C. For part A there are no subassemblies so all the subassembly BO CDFs are as follows:

Step 1

x	0	1	2	3	4	
P(#BO$\leq$x)	1.000	1.000	1.000	1.000	1.000	. . .

and multiplying by columns yields:

x	0	1	2	3	4	
P(#AWP$\leq$x)	1.000	1.000	1.000	1.000	1.000	. . .

Step 2, converting the AWP CDFs to a PF:

x	0	1	2	3	4	
P(#AWP=x)	1.000	0.000	0.000	0.000	0.000	. . .

Step 3.5, generating a Poisson AMU PF with an AMU = 1:

x	0	1	2	3	4	
P(#demands=x)	.3679	.3679	.1839	.0613	.0153	. . .

Step 4.5, combining the AMU PF and the AWP PF:

P(TOTMU=0) = P(AWP=0)xP(AMU=0)= 1.0 x .3679 = .3679.

P(TOTMU=1) = P(AWP=1)xP(AMU=0)+P(AWP=0)xP(AMU=1)

= (0 x .3679) + (1.0 x .3679)

= .3679

.

.

.

x	0	1	2	3	4	
P(#demands=x)	.3679	.3679	.1839	.0613	.0153	. . .

Step 6, converting this PF to a CDF:

x	0	1	2	3	4	
P(#demands≤x)	.3679	.7358	.9197	.9810	.9963	. . .

Step 7, adjusting this CDF for a stock level of 0 and a QPHA of 1 gives:

x	0	1	2	3	4	
P(#BO≤x)	.3679	.7358	.9197	.9810	.9963	. . .

Step 8, converting it to a BO PF:

x	0	1	2	3	4
P(#BO=x)	.3679	.3679	.1839	.0613	.0153 . . .

Step 9 Variance = 1.0

Mean or EBO = 1.0

Variance to mean ratio = 1.0

Step 10, the BO CDF in step 7 is transferred to parent part H for use in its calculations.

For part B the step results are as follows:

Step 1

x	0	1	2	3	4
P(#AWP≤x)	1.000	1.000	1.000	1.000	1.000 . . .

Step 2

x	0	1	2	3	4
P(#AWP=x)	1.000	0.000	0.000	0.000	0.000 . . .

Step 3.5, generating a Poisson AMU PF for AMU = 2:

x	0	1	2	3	4
P(#demands=x)	.1353	.2707	.2707	.1805	.0902 . . .

Step 4.5

x	0	1	2	3	4
P(#demands=x)	.1353	.2707	.2707	.1805	.0902 . . .

Step 6

x	0	1	2	3	4	
P(#demands≤x)	.1353	.4060	.6767	.8571	.9474	. . .

Step 7, adjusting for a stock level of 1 and a OPHA of 2:

x	0	1	2	3	4	
P(#BO≤x)	.4060	.8571	.9834	.9989	.9999	. . .

Step 8, converting to a PF:

x	0	1	2	3	4	
P(#BO=x)	.4060	.4511	.1263	.0155	.0011	. . .

Step 9 Variance = .5442

Mean or EBO = .7546

Variance to mean ratio = .7211

Step 10, transfer the BO CDF in step 7 to part H.

For part C

Step 1

x	0	1	2	3	4	
P(#BO≤x)	1.000	1.000	1.000	1.000	1.000	. . .

Step 2

x	0	1	2	3	4	
P(#BO=x)	1.000	0.000	0.000	0.000	0.000	. . .

Step 3.5, generating a Poisson AMU PF with AMU = 1:

x	0	1	2	3	4
P(#demands=x)	.3679	.3679	.1839	.0613	.0153 . . .

Step 4.5, TOTMU PF:

x	0	1	2	3	4
P(#demands=x)	.3679	.3679	.1839	.0613	.0153 . . .

Step 6, TOTMU CDF:

x	0	1	2	3	4
P(#demands≤x)	.3679	.7358	.9197	.9810	.9963 . . .

Step 7, adjusting the CDF for stock level of 0 and a QPHA of 2:

x	0	1	2	3	4
P(#BO≤x)	.3679	.9197	.9963	.9999	1.000 . . .

Step 8, BO PF:

x	0	1	2	3	4
P(#BO=x)	.3679	.518	.0766	.0036	.0001 . . .

Step 9 Variance = .3790

Mean or EBO = .7162

Variance to mean ratio = .5292

Step 10, transfer the BO CDF from step 7 to part H.

Now H, the type II part, is considered.

Step 1

x	0	1	2	3	4	
P(#A BO≤x)	.3679	.7358	.9197	.9810	.9963	. . .
P(#P BO≤x)	.4060	.8571	.9834	.9989	.9999	. . .
P(#C BO≤x)	.3679	.9197	.9963	.9999	1.000	. . .
P(#H AWP≤x)	.0549	.5800	.9012	.9799	.9963	. . .

Step 2, AWP PF:

x	0	1	2	3	4	
P(#AWP=x)	.0549	.5251	.3212	.0787	.0164	. . .

Step 3.5, generating a Poisson AMU PF with AMU = 1:

x	0	1	2	3	4	
P(#demands=x)	.3679	.3679	.1839	.0613	.0153	. . .

Step 4.5, TOTMU PF:

x	0	1	2	3	4	
P(#demands=x)	.0202	.2134	.3214	.2471	.1271	. . .

Step 6, TOTMU CDF:

x	0	1	2	3	4	
P(#demands≤x)	.0202	.2336	.5550	.8020	.9292	. . .

Step 7, adjusted CDF for stock level of 2 and a OPHA of 1:

x	0	1	2	3	4	
P(#BO≤x)	.5550	.8021	.9292	.9787	.9945	. . .

Step 8, BO PF:

x	0	1	2	3	4	
P(#BO=x)	.5550	.2471	.1271	.0496	.0157	. . .

Step 9 Variance = 1.059

Mean or EBO = .7422

Variance to mean ratio = 1.427

Step 10, transfer to parent of H, if there was one. There is no parent so the desired result is the EBO of part H.

In this simple parts hierarchy, both the EBO and the variance of the part H BO PF were increased by using the Poisson approximation. The EBO was increased 16% (from .7422 to .8612) and the variance was increased 36.6% (from 1.059 to 1.4466).

Sample computer output for the above example is included in appendix D for both programs.

Verification of Computer Models

To insure APPROX and EXACT function correctly the performances of each individual subprogram and the total program were evaluated.

The FACT function was checked to insure it produced the proper factorials over the entire range of positive

integers. The results of the GENPF subroutine were checked against tabulated Poisson values for several MUs. The conversion subroutines PF2CDF and CDF2PF, the adjustment subroutine SHIFTL, and the combining subroutine COMBYN were also checked for proper operation.

The overall operation of the programs was checked by comparing the computer output with hand calculated data for the simple sample problem in this chapter. The programs were also run for the examples given in various AFLC/XRS reports (Ref 9,15). All results agreed to at least four decimal places.

III Findings

The analysis was conducted in two parts. The first part was to determine what actually causes the errors in the Poisson approximation and to verify the conclusions drawn by constructing parts hierarchies and data sets which produced no error. The second part of the study examined the sensitivity of the errors to different paramenters.

Causes of Error

To determine the exact source of the approximation error many different data sets were run through both the APPROX and EXACT computer programs. Most cases exhibited some error but in some cases no error was observed. An in depth analysis resulted in the following conclusions.

The Poisson approximation initially preserves the correct mean but can distort the variance of the TOTMU PF. The distorted variance can subsequently induce errors for the following two reasons:

1. The approximation may assume a different variance for the TOTMU PF than it actually has, and when shifted for stock level and QPHA, it distorts the true mean (this error does not occur when the part has a stock level of zero and a QPHA of one because no shifting of the TOTMU PF takes place). This error is referred to as type 1

error in this report.

2. Also as a result of the Poisson approximation, the BO PFs may have more or less variance than they should (whether shifted of not). When the BO CDFs for SAs are transferred to their HA and multiplied column by column to get an AWP CDF for the HA, the mean is again distorted by the incorrect variance even if the means or EBOs were correct to start with. (This error does not occur when there is only one SA per HA, because no multiplying of BO CDFs by columns takes place. In this case, the HA AWP CDF is simply the SA BO CDF.) This error is referred to as type 2 error is this report.

It should also be pointed out that no error is ever induced for the parts at the very tips of the hierarchial tree. Because all parts are assumed to have a Poisson distributed AMU and the tip parts have no SAs, both APPROX and EXACT carry the same distributions up the tree to the first HAs. It is then at this level that the error is first introduced.

To show that the Poisson approximation preserves the correct mean but can distort the variance, consider the following two PFs as an example.

AN AMU PF

x	0	1	2
P(X=x)	.6	.3	.1

for this PF mean = .5 and variance =.45

AN AWP PF

x	0	1	2	3
P(X=x)	.1	.3	.4	.2

for this PF mean =1.7 and variance =.81

Using the APPROX method of combining PFs the means are added to get a mean of 2.2 from which the following Poisson PF is generated.

RESULTANT APPROX TOTMU PF

x	0	1	2	3	4	5	...
P(X=x)	.11	.24	.27	.20	.11	.05	...

for this PF mean =2.2 and variance =2.2

Using the EXACT method

$P(X=0) = .6 \times .1 = .06$

$P(X=1) = .3 \times .1 + .6 \times .3 = .21$

$P(X=2) = .1 \times .1 + .3 \times .3 + .6 \times .4 = .34$

$P(X=3) = .1 \times .3 + .3 \times .4 + .6 \times .2 = .27$

$P(X=4) = .1 \times .4 + .3 \times .2 = .10$

$P(X=5) = .1 \times .2 = .02$

which yields the following PF:

RESULTANT EXACT TOTMU PF

x	0	1	2	3	4	5
P(X=x)	.06	.21	.34	.27	.10	.02

for this PF mean =2.2 and variance =1.26

This example illustrates that the approximation gives the same mean but distorts the variance.

Working a similar example in general terms using the following two PFs:

PF 1

x	0	1	2
P(X=x)	A	B	C

mean = B + 2C

PF 2

x	0	1	2
$P(X=x)$	a	b	c

$$\text{mean} = b + 2c$$

Using the APPROX method of combining PFs the means are added to get a mean of $B + b + 2C + 2c$.

Using the EXACT method

$$P(X=0) = Aa$$

$$P(X=1) = Ab + Ba$$

$$P(X=2) = Ac + Bb + Ca$$

$$P(X=3) = Bc + Cb$$

$$P(X=4) = Cc$$

from which the following expression for the mean is obtained using expected value:

$$\text{Mean} = Ab + Ba + 2Ac + 2Bb + 2Ca + 3Bc + 3Cb + 4Cc$$

Since

$$A + B + C = 1 \quad \text{and} \quad a + b + c = 1$$

it follows that

$$A = 1 - B - C \quad \text{and} \quad a = 1 - b - c$$

Substituting these values into the above equation for the mean yields

$$\text{Mean} = B + b + 2C + 2c$$

The expressions for the APPROX and EXACT means are the same, demonstrating that the mean is initially preserved with the approximation.

<u>Type 1 Error</u>. To illustrate the type 1 error caused by the shifting of a TOTMU PF for stock level and OPHA, consider the demand distributions given below for a certain part. Both PFs have the same mean, but the first PF has a larger variance, such as could have been induced by the approximation.

APPROX TOTMU PF

x	0	1	2	3	4
P(X=x)	.2	.2	.3	.2	.1

mean = 1.8 variance = 1.56

EXACT TOTMU PF

x	0	1	2	3	4
P(X=x)	.1	.3	.35	.2	.05

mean = 1.8 variance = 1.06

If the distributions are adjusted for a stock level of one, the BO PFs are:

APPROX BO PF

x	0	1	2	3	4
P(X=x)	.4	.3	.2	.1	0

with mean = 1 and variance = 1

EXACT BO PF

x	0	1	2	3	4
P(X=x)	.4	.35	.2	.05	0

with mean = .9 and variance = .79

In this case shifting the distribution for a stock level of one results in the approximation giving a high mean and preserving the inflated variance.

If the original distributions are adjusted for a QPHA of two, the BO PFs are:

APPROX BO PF

x	0	1	2	3	4
P(X=x)	.2	.5	.3	0	0

with mean = 1.1 and variance = .49

EXACT BO PF

x	0	1	2	3	4
P(X=x)	.1	.65	.25	0	0

with mean = 1.15 and variance = .33

In this case, shifting the distribution for a QPHA of two results in the approximation giving a low mean and preserving the inflated variance.

If the original distributions are adjusted for a QPHA of two and a stock level of one, the BO PFs are:

APPROX BO PF

x	0	1	2	3	4
P(X=x)	.4	.5	.1	0	0

with mean = .7 and variance = .41

EXACT BO PF

x	0	1	2	3	4
P(X=x)	.4	.55	.05	0	0

with mean = .65 and variance = .33

In this final case, shifting the distribution results in the approximation giving a high mean and preserving the inflated variance.

This example shows that the distorted variance caused by the Poisson approximation induces an error in the mean, and that this error can be understated as well as overstated.

According to the explanation of type 1 error, any single-level parts hierarchy having an HA with a QPHA of 1 and a stock level of zero should have no error induced, regardless of the AMU of the HA or the AMUs, stock levels, or QPHAs of the subassemblies.

This hypothesis was tested with several different data sets, and there was never any error observed. However, when any stock level was included for the HA, or the QPHA was changed from one, errors resulted. Percent error was used to measure both mean and variance error and was calculated using the following formula:

$$\text{Percent error} = \frac{\text{APPROX results} - \text{EXACT results}}{\text{EXACT results}}$$

The data sets used and the results are included in appendix E.

<u>Type 2 Error</u>. To illustrate the type 2 error caused by transferring BO CDFs for SAs to their respective HA, con-

sider the demand distributions given below for a certain part. Both PFs have the same mean, but the first has a larger variance, such as could have been induced by the approximation.

APPROX BO PFs for 2 SAs

x	0	1	2	3
P(X=x)	.2	.4	.2	.2

with mean = 1.4, and variance = 1.04

EXACT BO PFs for 2 SAs

x	0	1	2	3
P(X=x)	.1	.5	.3	.1

with mean = 1.4, and variance = .64

Combining the two BO PFs in each case by

1. converting the PFs to CDFs
2. squaring the cumulative probabilities in each column
3. converting the resultant CDF back to a PF

yields the following AWP PFs for the HA:

APPROX AWP PF for HA

x	0	1	2	3
P(X=x)	.04	.32	.28	.36

with mean = 1.96, and variance = .84

EXACT AWP PF for HA

x	0	1	2	3
P(X=x)	.01	.35	.45	.19

with mean = 1.82, and variance = .55

This example shows that the inflated variance initially caused by the approximation can distort the mean in later calculations.

To test the type 2 error hypothesis, multiple indenture level hierarchies with one SA per HA on all but the lowest level were run. Again a QPHA of one and a stock level of zero for all parts were used to prevent any type 1 error. As expected, no error was recorded until more than one SA was associated with an HA, above the lowest level. The results of this test are included in appendix F.

It can therefore be concluded that whenever an HA has nonzero stock level or a QPHA not equal to one, or the hierarchy has more than one SA per HA above the lowest level in each branch, error may be induced.

Sensitivity Analysis

Sensitivity analysis was performed to study the effect of the following parameters on the Poisson approximation error:

- depth of indenture of the parts hierarchy
- SA to HA ratio for higher levels
- stock level of each part
- QPHA of each part

To check the effect of increasing depth of indenture, four different parts hierarchies were constructed, ranging from a "single-level" model to a "four-level" model. To insure that the change in the error was due only to increasing depth of indenture, the same basic part data was used in each additional level. The hierarchies, part data sets, and results are included in appendix C. A summary of the results is shown below.

ERROR OF THE MEAN USING
THE POISSON APPROXIMATION

Depth of Indenture	Set 1 % error	Set 2 % error
single-level	0	0
two-level	1.4	-7.6
three-level	3.3	-15.2
four-level	5.0	-22.3

The results suggest that the magnitude of the error in the mean for the Poisson approximation increases with increasing depth of indenture. This is expected since deeper indenture

increases the number of opportunities for type 1 and type 2 error to occur.

To evaluate the effect of SA to HA ratio, four different parts hierarchies were constructed. These hierarchies all had two levels of indenture and had constant SA to HA ratios ranging from two to five (for example, for a constant SA to HA ratio of three, each HA in the hierarchy would have three SAs). These parts hierarchies were all run for four different data sets. The hierarchies, part data sets, and results are included in appendix C. A summary of the results is shown below.

ERROR OF THE MEAN USING
THE POISSON APPROXIMATION

SA to HA Ratio	Set 1 % error	Set 2 % error	Set 3 %error	Set 4 %error
2	1.5	-7.6	2.8	-6.7
3	3.2	-6.9	6.2	-8.7
4	5.0	-5.6	8.6	-10.0
5	6.8	-3.8	10.5	-12.0

The results suggest that the error in the mean for the Poisson approximation can move in either direction with increasing SA to HA ratio. This is probably due to the fact that as the SA to HA ratio increases type 2 error tends to dominate. Depending on whether type 2 error is positively or negatively inclined, the error moves in the plus or minus direction.

To study the effect of different combinations of stock

level and QPHA on the error, the percent error was calculated for each combination using three different hierarchies. The actual results are included in appendix G, and a summary of the results is shown below.

For the first hierarchy and data set:

% ERROR OF THE MEAN USING THE POISSON APPROXIMATION

Stock Level	QPHA 1	QPHA 2	QPHA 3
0	4.9	6.8	5.0
1	1.6	6.0	5.9
2	-4.8	3.9	3.2
3	-15.2	1.3	2.8
4	-16.4	-.9	.3
5	-8.7	-.7	.06

For the second hierarchy and data set:

% ERROR OF THE MEAN USING THE POISSON APPROXIMATION

Stock Level	QPHA 1	QPHA 2	QPHA 3
0	1.4	2.2	1.8
1	1.2	2.1	1.7
2	.1	3.6	1.7
3	-1.5	.8	.9
4	-2.8	.1	.4

For the third hierarchy and data set:

% ERROR OF THE MEAN USING THE POISSON APPROXIMATION

Stock Level	QPHA 1	QPHA 2	QPHA 3
0	3.2	5.5	5.0
1	.5	4.3	4.2
2	-3.6	1.6	1.3
3	-5.2	.2	.6
4	-3.3	-.2	.1

The lack of monotonicity in the above results makes it difficult to arrive at specific conclusions. However, there are general trends present in all three cases which may shed some light on the problem. In all cases there seems to be:

a. A general decrease in positive error (not magnitude of error) with increasing stock level

b. A general increase in positive error (not magnitude of error) with increasing QPHA (in the low stock level/high QPHA sectors this does not hold).

c. A general cancelling of errors between the two factors as evidenced by the negative errors in the high stock and low QPHA sectors, and by the minimum errors occurring roughly along the indicated diagonals.

d. The errors of hierarchy two are generally less than the errors of the other two hierarchies.

This suggests that minimally indentured hierarchies induce less error than heavily indentured hierarchies.

IV Conclusions and Recommendations

The WARS function of building optimal WRM kits is extremely important. Stocking insufficient quantities could severely degrade the ability to successfully fight a war, while stocking excess quantities or unnecessary parts is very expensive in holding and opportunity costs. This is graphically portrayed by the fact that a WRSK for a single 24 aircraft A-10 squadron costs in excess of $8 million (Ref 17). The ability of WARS to build optimal kits is dependent on its ASSESS subroutine being able to accurately evaluate the performance of different parts packages in the marginal analysis process. Since ASSESS uses the Poisson approximation, understanding and evaluating its effect is of utmost importance.

Evaluating the effect of this approximation is extremely difficult because of the infinite number of possible hierarchy, MU, stock level, and QPHA combinations, and the interaction of the above parameters. Several specific hierarchies and data sets can easily be evaluated, but using induction to extrapolate this limited set of results to the general case is dangerous. Another uncertainty is the realism of the samples evaluated. All parts hierarchies used were fabricated since the part indenture relationships required are not currently available for operational air-

craft. Determining the effect of an approximation without actual data is a formidable problem, nevertheless, from this study the following conclusions were drawn:

Conclusions

(Note: Mean error means the error of the mean not average error.)

1. Mean error usually is induced when the Poisson approximation is used.

2. The approximation initially preserves the correct mean but can distort the variance of the part's demand probability function.

3. The distorted variance of a part's demand probability function can induce errors in the mean in subsequent computations.

4. The errors in the mean can be caused by two distinct processes:

   Type 1 - distortion of the mean when a PF is shifted for stock level or QPHA because the approximation may assume a different variance for the TOTMU PF than actual. (this error does not occur when the part has a stock level of zero and a QPHA of one because no shifting of the TOTMU PF takes place).

   Type 2 - distortion of the mean when the BO CDFs for SAs are transferred to their HA and multiplied column by column to get an AWP CDF for the HA, because the Poisson approximation distorts the variance of the BO PFs (This error does not occur when there is only one

SA per HA, because no multiplying of BO CDFs by columns takes place. In this case, the HA AWP CDF is simply the SA BO CDF.)

5. As a result of type 1 error, an error may be induced whenever stock level is more than zero or QPHA is greater than one for any parent part.

6. As a result of type 2 error, an error may also be induced whenever the parts hierarchy is such that an HA has more than one SA (excluding the bottom level).

7. If neither of the above conditions in conclusion 5 or conclusion 6 are present, no error will result.

8. Both mean and variance error vary in sign and magnitude from case to case.

9. Mean error seems to increase in magnitude with increasing depth of indenture.

10. Mean error seems to increase or decrease monotonically with increasing SA to HA ratio, depending on the sign of the original type 2 error.

11. The interaction of stock level and QPHA makes it difficult to arrive at specific conclusions. However, there are general trends present in all three cases which may shed some light on the problem. In all cases there seems to be:

a. A general decrease in positive error (not magnitude of error) with increasing stock level

b. A general increase in positive error (not magnitude of error) with increasing QPHA (in the low stock level/high QPHA sectors this does not hold).

c. A general cancelling of errors between the two factors as evidenced by the negative errors in the high stock and low QPHA sectors, and by the minimum errors occurring roughly along the indicated diagonals.

d. The errors of hierarchy two are generally less than the errors of the other two hierarchies. This suggests that minimally indentured hierarchies induce less error than heavily indentured hierarchies.

12. The error created may not be critical to WARS/ASSESS because:

a. There may be some cancelling of errors, both between type 1 and type 2 error as well as between the errors caused by the other approximations used in ASSESS.

b. Most actual aircraft parts hierarchies have a small percentage of their parts indentured to higher assemblies (Ref 8,18). Because of this characteristic, the approximation errors are diluted somewhat and tend to be smaller.

c. ASSESS is only used in the marginal analysis process to evaluate the increase in performance due to adding one part to a kit relative to another different part. Since both kits evaluated would be the same kit except for a few different parts, they would most likely suffer from similar errors and the methodology would probably still be able to pick the part, which if added, would increase performance most per dollar.

Recommendations

Although this study produced only a limited number of specific conclusions, it is hoped that the data computed and the general conclusions presented aid in the further understanding of this important approximation. To build upon these results, the following areas for further study are suggested:

1. In the future, when indentured information is available for operational aircraft, it would be

valuable to run actual hierarchies and parts data through the APPROX and EXACT programs and analyze the results.

2. Similar studies should be undertaken analyzing the effects of the other approximations used in WARS/ASSESS.

3. The combined effects of the three approximations should be examined in detail.

4. Since WARS will have the capability to use negative binomial distributions for each part's AMU distribution instead of Poisson, the study should be repeated for this distribution. The Poisson distribution is a special case of the negative binomial with the variance to mean ratio equal to one, so sensitivity to the variance to mean ratio should also be evaluated.

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Appendix A

Glossary (Ref 10)

AMU - A variable denoting the basic mean pipeline quantity for an item, excluding AWP. Also called "actual mu."

Assessment - In this report, assessment is the calculation of the expected NMCS, total expected backorders, and probability distribution of NMCS based on a specific stock level.

Availabiblity - The probability that a randomly selected item is not backordered.

AWP - A variable denoting the expected number of items "awaiting parts." This quantity is computed by the WARS assessment routine.

Backorder(BO) - An unfilled demand or shortage at base level.

BLSS kit - Stock of "Base Level Self-sufficiency Spares" designed to meet the additional requirements for spare parts, above normal peacetime requirements, when a war starts.

Cannable - Interchangeable with cannibalizable.

Cannibalizable item - An item which can be borrowed from an aircraft that already has a backorder to prevent a backorder on another aircraft. The extra maintenance time required to cannibalize a part is not considered, so an item which takes an excessive amount of time to cannibalize should be considered non-cannibalizable. Cannibalizable is synonymous with cannable.

CDF - Cumulative Distribution Function

Combined probabilities - This refers to calculating the probability distribution of the sum of probability distributions for two mutually exclusive random variables. For example, denoting two random variables by A and B, and the probability of x demands for A by $p(A = x)$, the probability distribution of the sum of demands for A and B is calculated as follows:

$$p(A+B=0) = p(A=0)p(B=0)$$

$$p(A+B=1) = p(A=0)p(B=1) + p(A=1)p(B=0)$$

$$\vdots$$

Cumulative probability - The probability of 0 through x demands, since only discrete, non- negative distributions are considered.

DFHP - Daily Flying Hour Program. The planned number of hours a unit will fly each day of a contingency.

Demand - Demand is interchangeable with "pipeline quantity" and failure.

Depth of Indenture - refers to the number of levels in a parts hierarchy. The more levels in a parts hierarchy, the greater the depth of indenture.

EBO - Expected number of backorders. If the item under consideration is the aircraft itself, EBO is equivalent to E[NMCS].

E[NMCS] - Expected number of NMCS aircraft. Also called E[NMC] in this report.

Failure - Failure is interchangeable with demand and "pipeline quantity."

HA - Higher Assembly.

Hierarchy diagram - A diagram showing the indenture relationships for all positions on the aircraft. For each item, this indenture relationship lists all higher assemblies that must be taken apart to reach this item, and all lower indenture items, or parts that can only be reached by taking this item apart.

Hierarchy position - The location of a part on the hierarchy diagram. A hierarchy position describes the location of only one stock number, but there may be more than one unit of the stock number in the hierarchy positon if its quantity per aircraft is larger than one. A stock number will have more than one hierarchy position (and will be called a non-unique item) if it is atttached to more than one parent item on the aircraft.

Higher assembly (HA) - An item that contains parts that can only be reached by taking apart this item. Sometimes called a "parent" with respect to immediately lower parts.

Imperfectly cannibalizable part (MCP) - An item that is considered cannbalizable but does not have the same quantity per higher assembly on all higher assemblies. Also referred to as "myopically cannibalizable".

Indenture - The concept of determining which items must be taken apart to reach a given part. An item is top indenture if it can be removed directly from the aircraft, regardless of where it is attached to the aircraft. A lower indenture part can be reached only by taking apart the item to which it is attached.

Individual probability - the probability of exactly x backorders (or other demands).

Kit - A stock of spare parts built for use at a single squadron, regardless of whether the stock consists of a WRSK, a BLSS, or normal peacetime supplies.

Lower indentured item - A part whcih can be reached only by taking apart an item to which this part is attached.

LRU - Line Replaceable Unit. An item which can be removed and replaced on the flightline. It contains one or more lower indenture items.

MA - Marginal Analysis.

MCP - Imperfectly cannibalizable, myopically cannibalizable, or imperfectly cannable part.

MU - pipeline quantity.

Myopic cannibalization - Same as imperfect cannibalization.

NCP - Non-Cannibalizable Part.

NMC - Not Mission Capable, used interchangeably with NMCS.

NMCS - Not Mission Capable due to Supply. - An aircraft missing parts is called NMCS, but a spare part can only be AWP if it has lower items backordered. Previously called NORS (Not Operationally Ready due to Supply).

Non-cannibalizable part (NCP) - An item that cannot be removed in a serviceable condition from one aircraft to prevent a backorder on another aircraft. While it may be physically possible to remove such an item, the item may be designated NC because the item would be damaged when it is removed or time to remove it is considered excessive.

Non-unique - A stock number that is used in more than one hierarchy position on an aircraft.

NSN - National Stock Number.

Parent item - The next higher assembly directly above an item in an indenture relationship.

PCP - Perfectly Cannibalizable Part

Perfectly cannibalizable part (PCP) - An item which is considered cannibalizable, and which has the same quantity per higher assembly on all higher assemblies containing the item.

PF - Probability function.

Pipeline quantity (MU) - The number of units of an item that have been removed from aircraft and are not yet in a serviceable condition at the base ready to be replaced on aircraft. This includes parts that are awaiting base maintenance, in base maintenance, awaiting parts at the base, awaiting or in transportation to the repair depot, awaiting maintenance at the depot, in maintenance at the depot, and awaiting or in transportation to the base. Demand, failure, or backorder may be used as a substitute for expected pipeline quantity.

Probability function (PF) - The set of the individual probability of each possible outcome for a discrete random variable.

QPA - Quantity Per Aircraft. This is the number of each item on one aircraft.

QPHA - Quantity Per Higher Assembly. This is the number of each item on its respective higher assembly.

SA - Subassembly.

SDO - Stock Due Out. The total number of parts on backorder.

SRU - Shop Replaceable Unit. An item which can be reached only by removing the LRU containing the item from the aircraft and disassembling the LRU.

Stock Level - the initial number of each part available to replace failed parts. For WRSK and BLSS it is the quantity included in the kit. For peacetime operation it is the quantity in base supply.

Stock number - Interchangeable with NSN.

Subassembly (SA) - A part indentured to a higher assembly.

TOTMU - A variable denoting the total expected pipeline

quantity for an item. It is equal to AMU + AWP.

Type (of an item) - The type of an item refers to its indenture relationship as follows:

4 = An LRU or engine attached directly to the aircraft.
3 = An LRU or module attached to a type 4 item.
2 = An LRU attached to a module.
1 = An SRU attached to any LRU.

UE - Unit equipment.

Unit equipment (UE) - The number of aircraft assigned to a unit.

Unique item - A stock number used in only one hierarchy position.

WARS - Wartime Assessment and Requirements System.

WRSK - War Readiness Spares Kit. A kit of spare parts to be taken with a unit in wartime, designed to support that unit for a specified period without outside maintenance support.

Appendix B

The Assessment Routine (Ref 9)

This section discusses the assessment routine as it is used in WARS ASSESS. First, some terms need to be defined:

Item types:

- Type I - an SRU assigned to an LRU.
- Type II - an LRU assigned to a module.
- Type III - an LRU or module assigned to an engine.
- Type IV - an LRU or engine assigned to an aircraft.

Stages one through four of the computation cover Type I through Type IV items respectively.

i = present item considered, in squadron S on day t.

It may be n stages denoted:

i_n = item which will not be cannibalized.

i_{pc} = item which is perfectly cannibalized.

i_{mc} = item which is imperfectly cannibalized.

$H(i)$ = higher assembly for item i.

j = quantity of i in an $H(i)$.

J = maximum quantity of i in any $H(i)$ = $J[H(i),S,t]$.

K = quantity of $H(i)$ considered at a certain step

in the process.

MU[(i,S,t)] = MU(i) = expected number of assets of item i accounted for at the end of day t in squadron S (basically equals the number of demands that must be filled by base stock to prevent backorders).

p(x,MU[i,S,t]) = p(x,MU[i]) = probability of x assets accounted for, given an expected number of MU[i,S,t].

A[i,S,t] = A[i] = number of assets of item i considered at the end of day t in squadron S (basically equals the number of serviceable items in base stock).

N[j,H(i),S,t] = N[j,H(i)] = number of H(i) that contain j of item i at the end of day t in squadron S.

N[≥j,H(i),S,t] = N[≥j,H(i)] = number of H(i) that contain between j and J of item i at the end of day t in squadron S.

N[H(i),S,t] = N[H(i)] = total number of higher assemblies considered in this run.

This may include some higher assemblies without item i, or N[0,H(i)]. Normally the, number of higher assemblies considered will be the number installed on all aircraft in the squadron.

EBO[i,S,t] = EBO[i] = expected number of backorders of item i in squadron S at the end of day t.

Q[H(i),S,t] = Q[H(i)] = QPHA of item i on H(i).

F[K,i,S,t] = F[K,i] = probability that K units or less of H(i) are missing item i at the end of day t in squadron S, assuming i is perfectly cannibalizable.

P[K,H(i),S,t] = P[K,H(i)] = probability that K units of H(i) are missing one or more i_{pc} items at the end of day t in squadron S.

q[i,S,t] = q[i] = expected proportion of units of H(i) that are not missng any of item i at the end of day t in squadron S, assuming i is non-cannibalizable.

C[K,i,S,t] = C[K,i] = probability that K units of H(i) are missing item i at the end of day t in squadron S, assuming that i is imperfectly cannibalizable, and where cannibalization consolidates the failures onto the smallest number of H(i).

M[H(i),S,t] = M[H(i)] = expected number of H(i) missing one or more of item i at the end of day t in squadron S, assuming i is imperfectly cannibalizable.

g[i,S,t] = g[i] = expected proportion of units of H(i) that are not missing any of item i at the end of day t in squadron S, assuming i is imperfectly cannibalizable.

b[H(i),S,t] = b[H(i)] = the probability that a random unit of H(i) is missing any i_n or i_{mc} items at the end of

day t in squadron S.

$B[K,H(i),S,t] = B[K,H(i)]$ = an approximate (binomial) probability that K units of H(i) are missing one or more i_n or i_{mc} at the end of day t in squadron S.

$\phi[K,H(i),S,t] = \phi[K,H(i)]$ = An approximate probability that K units of H(i) are down (or NMCS) at the end of day t in squadron S.

$E[H(i),S,t] = E[H(i)]$ = the expected number of units of H(i) down (or NMCS) at the end of day t in squadron S.

The procedure to be used in the example below is as follows: Steps a through d are performed once, at the beginning of the run, for all items. Each stage of the computation starts with step e.

Step a. Partition all items into the categories i_{pc}, i_n, and i_{mc}.

Step b. For each item, input A[i].

Step c. For each item, determine N[j,H(i)] for all appropriate j.

Step d. For each item, compute $N[H(i)] = \sum_j N[j,H(i)]$.

It is best to include j = 0 for those H(i) that could contain i but do not. Then, N[H(i)] will be the same for

all lower indentured items i with in the same H(i).

Step e. For each item in this stage, compute MU[i]. Actually, an "initial MU" will be calculated for each item at the beginning of the run. Ths "initial MU" is the expected pipeline quantity for the item, assuming no shortages of lower indentured items and no stock of this item.

In stages 2,3, and 4, the "final MU", or MU[i] as used here, will equal the "initial MU" plus E[H(i)], which is calculated for item i in step p of the previous stage. This means that MU[i] is equal to the expected pipeline quantity including shortages of lower indentured items. For example, supppose i in stage 2 and H(i) in stage 1 refer to item L1, and the "initial MU" for L1 is 0.5. This means that an average of 0.5 units of L1 have been removed from the aircraft and have not yet been returned to a serviceable condition at the base, excluding those units of L1 that are repaired but missing lower indentured parts, because of shortages. Assume that these lower indentured parts are SRUs S1 and S2 with MU[S1] = 0.5 and MU[S2] = 1.0. Also assume that in stage 1, E[L1] is found to be 1.3, meaning that the failures of an average of 0.5 + 1.0 = 1.5 units of S1 and S2 can be consolidated onto 1.3 units of L1. Assuming that the failures of L1, excluding those awaiting lower indentured parts, are not consolidated with the units of L1

that are awaiting lower indentured parts, MU[L1] = 0.5 + 1.3 = 1.8.

Step f. For each item in this stage, compute expected backorders, or

$$EBO[i] = \sum_{x=A[i]}^{\infty} (x - A[i])\ p(x,MU[i]) \qquad \langle 1 \rangle$$

Actually, the expected backorders do not need to be computed for cannibalizable items unless that data is needed for the objective function or for other evaluation purposes. A more efficient formula may be used for computer implementation:

$$EBO[i] = \sum_{x=A[i]}^{\infty} (1 - p(x,MU[i])$$

In this formula, the upper limit may be replaced by A[i] plus the number of parts installed on all the aircraft in the squadron.

Step g. For each i_{pc} item, compute:

$$F[K,i] = \sum_{x=0}^{A[i] + K\cdot Q[H(i)]} p(x,MU[i]) \qquad \langle 2 \rangle$$

for K = 0,1,2,...,(N[H(i) - N[0,H(i)])

Step h. For each i_n item, compute:

$$q[i] = \sum_{j} \left[\frac{N[j,H(i)]}{N[H(i)]} \left(1 - \frac{EBO[i]}{\sum_{j} j \cdot N[j,H(i)]} \right) \right]^{j} \qquad \langle 3 \rangle$$

Step i. For each i_{mc} item compute C[K(i)] for all K = 0,1,2,..., (N[H(i)] - N[0,H(i)]):

for K$\leqslant$ N[J,H(i)],

$$C[K,i] = \sum_{x=0}^{A[i] + J \cdot K} p(x,MU[i])$$

for N[J,H(i)] < K $\leqslant$ N[$\geqslant$J-1, H(i)],

$$C[K,i] = \sum_{x=0}^{A[i] + J \cdot N[J,H(i)] + (J-1)(K-N[J,H(i)])} p(x,MU[i])$$

for N[$\geqslant$J-1,H(i)] < K $\leqslant$ N[$\geqslant$J-2,H(i)],

$$C[K,i] = \sum_{x=0}^{A[i]+J\cdot N[J,H(i)]+(J-1)N[J-1,H(i)]+(J-2)(K-N[J,H(i)])} p(x,MU[i])$$

.

.

.

for N[$\geqslant$2,H(i)] < K $\leqslant$ =N[$\geqslant$1,H(i)],

$$C[K,i] = \sum_{x=0}^{A[i]+J\cdot N[J,H(i)]+...+2\ N[2,H(i)]+1(K-N[\geqslant 2,H(i)])} p(x,MU[i])$$

In other words, this means to calculate the cumulative probability of less than or equal to K units of H(i) down

for all relevant K, assuming that cannibalization consolidates the failures onto the smallest possible number of H(i). This happens by cannibalizing in order from the H(i) containing the most i to the least i.

Then,

$$M[H(i)] = \sum_{K=1}^{N[H(i) - N[0,H(i)]} K(C[K,i] - C[K-1,i])$$

Compute:

$$g[i] = 1 - M[H(i) / N[H(i)] \qquad \langle 4\rangle$$

Step j. For all i_n items in each H(i), compute:

$$\prod_{i_n} \text{[illegible]}_{n}] \qquad \langle 5\rangle$$

The q values are from formula <3>.

Step k. For all i_{mc} items in each H(i) compute:

$$\prod_{i_{mc}} g[i_{mc}] \qquad \langle 6\rangle$$

The g values are from formula <4>.

Step l. For each H(i) compute:

$$b[H(i)] = 1 - \prod_{i_n} q[i_n] \cdot \prod_{i_{mc}} g[i_{mc}]$$

Step m. For each H(i), compute binomial probabilities for all K = 0,1,2,...,N[H(i)]:

$$B[K,H(i)] = \sum_{x=0}^{K} \left[\frac{N[H(i)]!}{(N[H(i)]-x)!\,x!}\right] b[H(i)]^{x} \, (1-b[H(i)])^{N[H(i)]-x} \qquad \langle 7\rangle$$

Step n. For all K = 0,1,2,...,N[H(i)] compute for each H(i):

$$P[K,H(i)] = \prod_{i_{pc}} F[K,i_{pc}] \qquad \langle 8\rangle$$

The F values are from formula <2>.

Step o. For all K = 0,1,2,...N[H(i)] compute for each H(i):

$$\phi[K,H(i)] = (B[K,H(i))(P[K,H(i)]) \qquad \langle 9\rangle$$

Step p. For each H(i) compute:

$$E[H(i)] = \sum_{K=1}^{N[H(i)]} K(\phi[K,H(i)] - \phi[K-1,H(i)]) \qquad \langle 10\rangle$$

Formula <10> is the number of H(i) expected to be down due to lack of lower indentured items. Actually, in an

AD-A139 491 THE EFFECT OF THE POISSON APPROXIMATION ON THE WARTIME ASSESSMENT AND REQ..(U) AIR FORCE INST OF TECH WRIGHT-PATTERSON AFB OH SCHOOL OF ENGI.. 2/2

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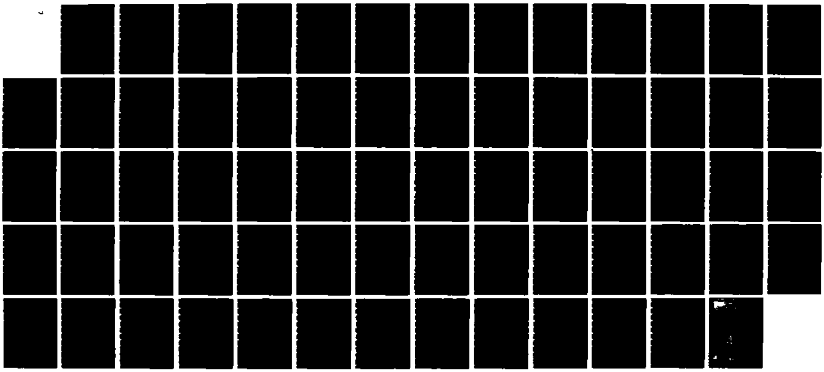

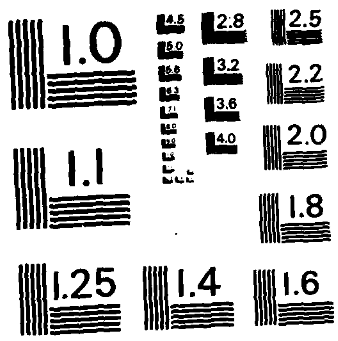

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operational implementation, a more efficient formula would be used:

$$E[H(i)] = N[H(i)] - \sum_{K=0}^{N[H(i)]-1} \phi[K,H(i)]$$

At the end of stage 4, the value of E[H(i)] is the expected number of NMCS aircraft, since H(i) is now the aircraft. If desired, the total number of expected backorders is:

$\sum_{i}$ EBO[i]. All results apply to squadron S at time t.

Appendix C

Program Listings

Note: The programs are written in FORTRAN 77 and can be run for any data set by simply changing the DATA statement and the NPARTS parameter throughout the programs. A list of computer variables used follows the two program listings.

```
      PROGRAM APPROX

**********************************************************************
* THIS PROGRAM CALCULATES THE APPROXIMATE E[NMC] AND INTERMEDIATE *
* DEMAND AND BACKORDER DISTRIBUTIONS FOR A SPECIFIED PARTS        *
* HIERARCHY AND STOCK LEVEL                                       *
**********************************************************************

      INTEGER  P,L,C,TYPE,BLOCK,NPARTS,DISTL
      CHARACTER*40  MSG(1:20)
      PARAMETER  (NPARTS=3,DISTL=27,BLOCK=6*(NPARTS+1)*(DISTL+1))
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      DATA  ((A(P,1,C),C=1,7),P=0,NPARTS)/
     +  5,1,1,0,0,0,0,
     +  4,3,1,1,2,0,1,
     +  3,0,1,1,0,1,1,
     +  3,0,2,2,1,1,2,
     +  3,0,2,1,0,1,3/
     +  (((A(P,L,C),P=0,NPARTS),L=2,7),C=0,DISTL)/
     +  BLOCK*1.0/
     +  MSG/'0/TYPE/#SA/QPHA/MU/SL/HA#/PUP','SA 1 BO CDF',
     +  'SA 2 BO CDF','SA 3 BO CDF','SA 4 BO CDF','SA 5 BO CDF',
     +  'AWP CDF','AWP PF','VAR/AWP/VAR TO MEAN RATIO/TOTMU',
     +    'TOTMU PF','TOTMU CDF',
     +  'BO CDF','BO PF','VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP',
     +  '***** ERROR *****','IMPROPER PART TYPE IN DATA',
     +  'NEGATIVE VALUE IN DATA','> 5 SA PER PARENT PART',
     +  'HA# > NPARTS','PUP > PARENT''S #SA'/


* ECHO PRINT DATA AND ERROR CHECK
      PRINT*,'INPUT DATA'
      PRINT*
      PRINT*,'PART  TYPE   #SA  QPHA    MU    SL   HA#   PUP'
      DO 10 P=0,NPARTS
          WRITE(*,5)P,(A(P,1,C),C=1,7)
    5     FORMAT(I5,7F6.1)
          IF (A(P,1,1).GT.5.0 .OR. A(P,1,1).LT.1.0) THEN
           PRINT*,MSG(15)
```

```
          PRINT*,MSG(16)
         END IF
         IF (A(P,1,2).GT.5.0) THEN
          PRINT*,MSG(15)
          PRINT*,MSG(18)
         END IF
         IF (A(P,1,6).GT.NPARTS) THEN
          PRINT*,MSG(15)
          PRINT*,MSG(19)
         END IF
         IF (A(P,1,7).GT.A(A(P,1,6),1,2)) THEN
          PRINT*,MSG(15)
          PRINT*,MSG(20)
         END IF
         DO 20 C=1,7
          IF (A(P,1,C).LT.0.0) THEN
              PRINT*,MSG(15)
              PRINT*,MSG(17)
          END IF
   20       CONTINUE
   10   CONTINUE
      PRINT*

* STEP THROUGH INDENTURE LEVELS FROM LOWEST TO HIGHEST LEVEL
      DO 40 TYPE=1,5

* FOR EACH PART, IF IT IS THE TYPE BEING CONSIDERED, PERFORM CALCULATIONS
          DO 50 P=0,NPARTS
           IF (A(P,1,1).EQ.TYPE) THEN

* COMPUTES AWP CDF FOR PART
               DO 60 C=0,DISTL
                DO 70 L=2,6
                    A(P,7,C)=A(P,7,C)*A(P,L,C)
   70                CONTINUE
   60                CONTINUE

* CONVERT AWP CDF TO AWP PF
               CALL CDF2PF(P,7,8)

* COMPUTE AWP AND VARIANCE; ADD AWP TO AMU
* IF PART IS THE AIRCRAFT THEN CALCULATE AWP AND
* AWP VARIANCE AND GO TO THE END OF THE LOOP
               IF (TYPE.EQ.5) THEN
                CALL CALEBO(P,8,14,1)
                CALL CALVAR(P,8,14,0)
                GO TO 40
               ELSE
                CALL CALEBO(P,8,9,1)
                A(P,14,15)=A(P,9,1)
                IF (A(P,9,1).GT.0.0) THEN
                    CALL CALVAR(P,8,9,0)
```

```
                  A(P,14,14)=A(P,9,0)
                  A(P,14,16)=A(P,9,2)
              END IF
              A(P,9,3)=A(P,9,1)+A(P,1,4)
             END IF

* GENERATE TOTMU DISTRIBUTION FROM CALCULATED TOTMU
* COMPUTE TOTMU MEAN AND VARIANCE
             CALL GENPF(P,A(P,9,3),10)
             CALL CALEBO(P,10,14,8)
             IF (A(P,14,8).GT.0.0) THEN
                 CALL CALVAR(P,10,14,7)
             END IF

* CONVERT TOTMU PF TO TOTMU CDF
             CALL PF2CDF(P,10,11)

* ADJUST TOTMU CDF FOR STOCK LEVEL AND QPHA TO GET BACKORDER CDF
             CALL SHIFTL(P,11,P,12,INT(A(P,1,5)),
     +                INT(A(P,1,3)))

* CONVERT BACKORDER CDF TO BACKORDER PF
             CALL CDF2PF(P,12,13)

* CALCULATE EXPECTED VALUE OF BACKORDER PF
             CALL CALEBO(P,13,14,1)

* CALCULATE VARIANCE AND VARIANCE TO MEAN RATIO OF BACKORDER PF
             CALL CALVAR(P,13,14,0)

* TRANSFER BACKORDER CDF TO PARENT RECORD
             CALL SHIFTL(P,12,INT(A(P,1,6)),
     +                1+INT(A(P,1,7)),0,1)
            END IF
   50        CONTINUE
   40   CONTINUE

* PRINT E[NMC] AND PFS
      PRINT*
      PRINT*,'APPROX RESULTS'
      DO 80 P=0,NPARTS
          PRINT*
          PRINT*,'PART NO. ',P,' * * * * * * * * * *',
     +             '* * * * * * * * * * * * * * * * *'
          DO 90 L=2,14
           PRINT*
           PRINT*,MSG(L)
           PRINT 95,(A(P,L,C),C=0,DISTL)
   90        CONTINUE
   95      FORMAT(7F9.5)
   80   CONTINUE
      END
```

```
**************************************************************************

      FUNCTION FACT(Y)
* THIS FUNCTION CALCULATES THE FACTORIAL OF A GIVEN ARGUMENT
      INTEGER  Y,K
      REAL  FACT
      IF (Y.LT.0) THEN
         PRINT*,'ERROR-TRIED TO TAKE FACTORIAL OF A NEGITIVE NUMBER'
         END IF
      FACT=1
      DO 100 K=1,Y
          FACT=FACT*K
  100    CONTINUE
      END

**************************************************************************

      SUBROUTINE PF2CDF(P,LF,LT)
* THIS SUBROUTINE CONVERTS A PF TO A CDF
      INTEGER  P,LF,LT,K,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      A(P,LT,0)=A(P,LF,0)
      DO 110 K=1,DISTL
          A(P,LT,K)=A(P,LF,K)+A(P,LT,K-1)
  110    CONTINUE
      END

**************************************************************************

      SUBROUTINE CDF2PF(P,LF,LT)
* THIS SUBROUTINE CONVERTS A CDF TO A PF
      INTEGER  P,LF,LT,K,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      A(P,LT,0)=A(P,LF,0)
      DO 120 K=1,DISTL
           A(P,LT,K)=A(P,LF,K)-A(P,LF,K-1)
  120    CONTINUE
      END

**************************************************************************

      SUBROUTINE GENPF(P,MU,LT)
* THIS SUBROUTINE GENERATES A POISSON PROBABILITY FUNCTION
      INTEGER  P,LT,Y,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  MU,FACT
      REAL  A(0:NPARTS,1:14,0:DISTL)
```

```
      COMMON  A
      DO 130 Y=0,DISTL
          A(P,LT,Y)=MU**Y*EXP(-MU)/FACT(Y)
 130   CONTINUE
      END

*************************************************************************

      SUBROUTINE SHIFTL(PF,LF,PT,LT,SL,QP)
* THIS SUBROUTINE ADJUSTS A DEMAND CDF FOR STOCK LEVEL AND
* QPHA TO GET A BACKORDER CDF
      INTEGER  PF,LF,PT,LT,SL,QP,K,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      DO 140 K=0,INT((DISTL-SL)/QP)
          A(PT,LT,K)=A(PF,LF,SL+K*QP)
 140   CONTINUE
      DO 145 K=INT((DISTL-SL)/QP)+1,DISTL
          A(PT,LT,K)=1.0
 145   CONTINUE
      END

*************************************************************************

      SUBROUTINE CALEBO(P,LF,LT,CT)
* THIS SUBROUTINE CALCULATES EXPECTED BACKORDER FROM A PF
      INTEGER  P,LF,LT,CT,K,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  EBO
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      EBO=0.0
      DO 150 K=0,DISTL
          EBO=EBO+K*A(P,LF,K)
 150   CONTINUE
      A(P,LT,CT)=EBO
      END

*************************************************************************

      SUBROUTINE COMBYN(P,LF1,LF2,LT)
* THIS SUBROUTINE COMBINES TWO MUTUALLY EXCLUSIVE PFS TO GET
* A RESULTANT PF
      INTEGER  N,C,P,LF1,LF2,LT,N,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      DO 160 C=0,DISTL
          A(P,LT,C)=0.0
          DO 160 N=0,C
           A(P,LT,C)=A(P,LT,C)+A(P,LF1,N)*A(P,LF2,C-N)
```

```
 160        CONTINUE
    END

**********************************************************************

    SUBROUTINE CALVAR(P,LF,LT,CT)
* THIS SUBROUTINE CALCULATES THE VARIANCE AND
* VARIANCE TO MEAN RATIO OF A BACKORDER PF
    INTEGER  P,LF,LT,CT,K,DISTL,NPARTS
    PARAMETER  (NPARTS=3,DISTL=27)
    REAL  SUM
    REAL  A(0:NPARTS,1:14,0:DISTL)
    COMMON  A
    SUM=0.0
    DO 170 K=0,DISTL
        SUM=SUM+((K-A(P,LT,CT+1))**2)*A(P,LF,K)
 170   CONTINUE
    A(P,LT,CT)=SUM
    A(P,LT,CT+2)=A(P,LT,CT)/A(P,LT,CT+1)
    END

**********************************************************************
```

```
      PROGRAM EXACT

***********************************************************************
* THIS PROGRAM CALCULATES THE EXACT E[NMC] AND INTERMEDIATE           *
* DEMAND AND BACKORDER DISTRIBUTIONS FOR A SPECIFIED PARTS            *
* HIERARCHY AND STOCK LEVEL                                           *
***********************************************************************

      INTEGER  P,L,C,TYPE,BLOCK,NPARTS,DISTL
      CHARACTER*40  MSG(1:20)
      PARAMETER  (NPARTS=3,DISTL=27,BLOCK=6*(NPARTS+1)*(DISTL+1))
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      DATA  ((A(P,1,C),C=1,7),P=0,NPARTS)/
      +  5,1,1,0,0,0,0,
      +  4,3,1,1,2,0,1,
      +  3,0,1,1,0,1,1,
      +  3,0,2,2,1,1,2,
      +  3,0,2,1,0,1,3/
      +  (((A(P,L,C),P=0,NPARTS),L=2,7),C=0,DISTL)/
      +  BLOCK*1.0/
      +  MSG/'0/TYPE/#SA/QPHA/MU/SL/HA#/PUP','SA 1 BO CDF',
      +  'SA 2 BO CDF','SA 3 BO CDF','SA 4 BO CDF','SA 5 BO CDF',
      +  'AWP CDF','AWP PF','AMU PF','TOTMU PF','TOTMU CDF',
      +  'BO CDF','BO PF','VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP',
      +  '***** ERROR *****','IMPROPER PART TYPE IN DATA',
      +  'NEGATIVE VALUE IN DATA','> 5 SA PER PARENT PART',
      +  'HA# > NPARTS','PUP > PARENT''S #SA'/


* ECHO PRINT DATA AND ERROR CHECK
      PRINT*,'INPUT DATA'
      PRINT*
      PRINT*,'PART  TYPE   #SA  QPHA    MU    SL    HA#    PUP'
      DO 10 P=0,NPARTS
          WRITE(*,5)P,(A(P,1,C),C=1,7)
    5     FORMAT(I5,7F6.1)
          IF (A(P,1,1).GT.5.0 .OR. A(P,1,1).LT.1.0) THEN
           PRINT*,MSG(15)
           PRINT*,MSG(16)
          END IF
          IF (A(P,1,2).GT.5.0) THEN
           PRINT*,MSG(15)
           PRINT*,MSG(18)
          END IF
          IF (A(P,1,6).GT.NPARTS) THEN
           PRINT*,MSG(15)
           PRINT*,MSG(19)
          END IF
```

```
          IF (A(P,1,7).GT.A(A(P,1,6),1,2)) THEN
           PRINT*,MSG(15)
           PRINT*,MSG(20)
          END IF
          DO 20 C=1,7
           IF (A(P,1,C).LT.0.0) THEN
               PRINT*,MSG(15)
               PRINT*,MSG(17)
           END IF
   20        CONTINUE
   10     CONTINUE
      PRINT*

* GENERATE AMU DISTRIBUTION FOR EACH PART FROM PIPELINE QUANTITY
      DO 30 P=1,NPARTS
          CALL GENPF(P,A(P,1,4),9)
   30     CONTINUE

* STEP THROUGH INDENTURE LEVELS FROM LOWEST TO HIGHEST LEVEL
      DO 40 TYPE=1,5

* FOR EACH PART, IF IT IS THE TYPE BEING CONSIDERED, PERFORM CALCULATIONS
          DO 50 P=0,NPARTS
           IF (A(P,1,1).EQ.TYPE) THEN

* COMPUTES AWP CDF FOR PART
               DO 60 C=0,DISTL
                DO 70 L=2,6
                    A(P,7,C)=A(P,7,C)*A(P,L,C)
   70                CONTINUE
   60                CONTINUE

* CONVERT AWP CDF TO AWP PF
               CALL CDF2PF(P,7,8)

* CALCULATE MEAN AWP AND AWP VARIANCE
* IF PART IS THE AIRCRAFT THEN GO TO THE END OF THE LOOP
               IF (TYPE.EQ.5) THEN
                CALL CALEBO(P,8,14,1)
                CALL CALVAR(P,8,14,0)
                GO TO 40
               ELSE
                CALL CALEBO(P,8,14,15)
                IF (A(P,14,15).GT.0.0) THEN
                    CALL CALVAR(P,8,14,14)
                END IF
               END IF
* COMBINE AWP PF AND AMU PF TO GET TOTMU PF
* COMPUTE MEAN AND VARIANCE OF TOTMU PF
               CALL COMBYN(P,8,9,10)
               CALL CALEBO(P,10,14,8)
               IF (A(P,14,8).GT.0.0) THEN
```

```
              CALL CALVAR(P,10,14,7)
             END IF

* CONVERT TOTMU PF TO TOTMU CDF
             CALL PF2CDF(P,10,11)

* ADJUST TOTMU CDF FOR STOCK LEVEL AND QPHA TO GET BACKORDER CDF
             CALL SHIFTL(P,11,P,12,INT(A(P,1,5)),
    +               INT(A(P,1,3)))

* CONVERT BACKORDER CDF TO BACKORDER PF
             CALL CDF2PF(P,12,13)

* CALCULATE EXPECTED VALUE OF BACKORDER PF
             CALL CALEBO(P,13,14,1)

* CALCULATE VARIANCE AND VARIANCE TO MEAN RATIO OF BACKORDER PF
             CALL CALVAR(P,13,14,0)

* TRANSFER BACKORDER CDF TO PARENT RECORD
             CALL SHIFTL(P,12,INT(A(P,1,6)),
    +               1+INT(A(P,1,7)),0,1)
            END IF
   50        CONTINUE
   40   CONTINUE

* PRINT E[NMC] AND PFS
    PRINT*
    PRINT*,'EXACT RESULTS'
    DO 80 P=0,NPARTS
        PRINT*
        PRINT*,'PART NO. ',P,' * * * * * * * * * * *',
    +          '* * * * * * * * * * * * * * * *'
        DO 90 L=2,14
         PRINT*
         PRINT*,MSG(L)
         PRINT 95,(A(P,L,C),C=0,DISTL)
   90      CONTINUE
   95   FORMAT(7F9.5)
   80   CONTINUE
    END

************************************************************************

    FUNCTION FACT(Y)
* THIS FUNCTION CALCULATES THE FACTORIAL OF A GIVEN ARGUMENT
    INTEGER  Y,K
    REAL  FACT
    IF (Y.LT.0) THEN
        PRINT*,'ERROR-TRIED TO TAKE FACTORIAL OF A NEGITIVE NUMBER'
    END IF
    FACT=1
```

```
      DO 100 K=1,Y
          FACT=FACT*K
  100   CONTINUE
      END

***********************************************************************

      SUBROUTINE PF2CDF(P,LF,LT)
* THIS SUBROUTINE CONVERTS A PF TO A CDF
      INTEGER  P,LF,LT,K,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      A(P,LT,0)=A(P,LF,0)
      DO 110 K=1,DISTL
          A(P,LT,K)=A(P,LF,K)+A(P,LT,K-1)
  110   CONTINUE
      END

***********************************************************************

      SUBROUTINE CDF2PF(P,LF,LT)
* THIS SUBROUTINE CONVERTS A CDF TO A PF
      INTEGER  P,LF,LT,K,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      A(P,LT,0)=A(P,LF,0)
      DO 120 K=1,DISTL
          A(P,LT,K)=A(P,LF,K)-A(P,LF,K-1)
  120   CONTINUE
      END

***********************************************************************

      SUBROUTINE GENPF(P,MU,LT)
* THIS SUBROUTINE GENERATES A POISSON PROBABILITY FUNCTION
      INTEGER  P,LT,Y,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  MU,FACT
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      DO 130 Y=0,DISTL
          A(P,LT,Y)=MU**Y*EXP(-MU)/FACT(Y)
  130   CONTINUE
      END

***********************************************************************

      SUBROUTINE SHIFTL(PF,LF,PT,LT,SL,QP)
* THIS SUBROUTINE ADJUSTS A DEMAND CDF FOR STOCK LEVEL AND
* QPHA TO GET A BACKORDER CDF
```

```
      INTEGER  PF,LF,PT,LT,SL,QP,K,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      DO 140 K=0,INT((DISTL-SL)/QP)
         A(PT,LT,K)=A(PF,LF,SL+K*QP)
  140   CONTINUE
      DO 145 K=INT((DISTL-SL)/QP)+1,DISTL
         A(PT,LT,K)=1.0
  145   CONTINUE
      END

**********************************************************************

      SUBROUTINE CALEBO(P,LF,LT,CT)
* THIS SUBROUTINE CALCULATES EXPECTED BACKORDER FROM A PF
      INTEGER  P,LF,LT,CT,K,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  EBO
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      EBO=0.0
      DO 150 K=0,DISTL
         EBO=EBO+K*A(P,LF,K)
  150   CONTINUE
      A(P,LT,CT)=EBO
      END

**********************************************************************

      SUBROUTINE COMBYN(P,LF1,LF2,LT)
* THIS SUBROUTINE COMBINES TWO MUTUALLY EXCLUSIVE PFS TO GET
* A RESULTANT PF
      INTEGER  N,C,P,LF1,LF2,LT,N,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      DO 160 C=0,DISTL
         A(P,LT,C)=0.0
         DO 160 N=0,C
          A(P,LT,C)=A(P,LT,C)+A(P,LF1,N)*A(P,LF2,C-N)
  160       CONTINUE
      END

**********************************************************************

      SUBROUTINE CALVAR(P,LF,LT,CT)
* THIS SUBROUTINE CALCULATES THE VARIANCE AND
* VARIANCE TO MEAN RATIO OF A BACKORDER PF
      INTEGER  P,LF,LT,CT,K,DISTL,NPARTS
      PARAMETER  (NPARTS=3,DISTL=27)
      REAL  SUM
```

```
      REAL  A(0:NPARTS,1:14,0:DISTL)
      COMMON  A
      SUM=0.0
      DO 170 K=0,DISTL
          SUM=SUM+((K-A(P,LT,CT+1))**2)*A(P,LF,K)
  170   CONTINUE
      A(P,LT,CT)=SUM
      A(P,LT,CT+2)=A(P,LT,CT)/A(P,LT,CT+1)
      END
```

****

Variables Used in the Computer Programs

A - data and probability array.

APPROX - program which calculates approximate E[NMC].

BLOCK - computed parameter.

BO - output label denoting backorder.

C - indexing variable for the column number in the A array which calculates the EBO from a probability function (PF).

CALEBO - subroutine which calculates the EBO from a PF.

CALVAR - subroutine which calculates variance and variance to mean ratio of a backorder PF.

CDF2PF - subroutine which converts a cumulative distribution function (CDF) to a PF.

COMBYN - subroutine which combines two mutually exclusive PFs to get a resultant PF.

DISTL - length of probability distributions.

DLPAR - distribution length paramenter.

EBO - expected backorder.

EXACT - program which calculates exact E[NMC].

FACT - function which calculates the factorial of a number.

GENPF - subroutine which generates a Poisson PF.

L - indexing variable for the line number in the A array.

LF - indexing variable indicating the line number of array A from which a probability distribution is taken.

LF1 - indexing variable of array A indicating the line from which the first of two mutually exclusive PFs is taken.

LF2 - indexing variable of array A indicating the line from which the second of two mutually exclusive PFs is taken.

LT - indexing variable indicating the line of array A to which a resultant probability distribution is assigned.

MSC - message and heading array.

MU - parameter from which the Poisson distribution is generated (as an output label it indicates the pipeline quantity for an item).

NPARTS - the number of different parts on the aircraft hierarchy.

P - indexing variable for the part number in the A array.

PF - indexing variable for array A indicating the part from which a probability distribution is taken.

PF2CDF - subroutine which converts a PF to a CDF.

PT - indexing variable for array A indicating the part to which the resultant probability distribution is assigned.

PUP - position of a subassembly under a parent item, for example, if a parent item has two subassemblies(SA), the first SA will have a PUP of 1 and the second SA will have a PUP of 2.

QP - QPHA for a specific part.

SHIFTL - subroutine which adjusts a demand CDF for stock level and QPHA to get a backorder CDF.

SL - stock level for a specific part.

SUM - computational variable used in calculating variance.

TYPE - part type.

#SA - output label indicating the number of different subassemblies on an item.

Appendix D

Sample Results

Note: E[NMC] is the boxed figure at the end of the data for part number 0.

Approx Results for Sample Problem

INPUT DATA

PART	TYPE	#SA	QPHA	MU	SL	HA#	PUP
0	5.0	1.0	1.0	.0	.0	.0	.0
1	4.0	3.0	1.0	1.0	2.0	.0	1.0
2	3.0	.0	1.0	1.0	.0	1.0	1.0
3	3.0	.0	2.0	2.0	1.0	1.0	2.0
4	3.0	.0	2.0	1.0	.0	1.0	3.0

APPROX RESULTS

PART NO. 0 *

SA 1 BO CDF

.54678	.76004	.89272	.95875	.98613	.99587	.99890
.99973	.99994	.99999	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 2 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 3 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 4 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 5 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP CDF

.54678	.76004	.89272	.95875	.98613	.99587	.99890
.99973	.99994	.99999	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP PF

.54678	.21326	.13267	.06603	.02739	.00974	.00303
.00084	.00021	.00005	.00001	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/AWP/VAR TO MEAN RATIO/TOTMU

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU PF

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU CDF

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

BO CDF

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

BO PF

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP

1.44662	.86116	1.67985	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

PART NO. 1 *

SA 1 BO CDF

.36788	.73576	.91970	.98101	.99634	.99941	.99992
.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 2 BO CDF

.40601	.85712	.98344	.99890	.99995	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 3 BO CDF

.36788	.91970	.99634	.99992	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 4 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 5 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP CDF

.05495	.58000	.90116	.97985	.99629	.99940	.99992
.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP PF

.05495	.52505	.32116	.07870	.01644	.00311	.00051
.00007	.00001	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/AWP/VAR TO MEAN RATIO/TOTMU

.66601	1.48845	.44745	2.48845	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU PF

.08304	.20664	.25710	.21326	.13267	.06603	.02739
.00974	.00303	.00084	.00021	.00005	.00001	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU CDF

.08304	.28968	.54678	.76004	.89272	.95875	.98613
.99587	.99890	.99973	.99994	.99999	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO CDF

.54678	.76004	.89272	.95875	.98613	.99587	.99890
.99973	.99994	.99999	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO PF

.54678	.21326	.13267	.06603	.02739	.00974	.00303
.00084	.00021	.00005	.00001	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP

1.44662	.86116	1.67985	.00000	.00000	.00000	.00000
2.48845	2.48845	1.00000	.00000	.00000	.00000	.00000
.66601	1.48845	.44745	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

PART NO. 2 *

SA 1 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 2 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 3 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 4 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 5 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP PF

1.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/AWP/VAR TO MEAN RATIO/TOTMU

.00000	.00000	.00000	1.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU PF

.36788	.36788	.18394	.06131	.01533	.00307	.00051
.00007	.00001	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU CDF

.36788	.73576	.91970	.98101	.99634	.99941	.99992
.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO CDF

.36788	.73576	.91970	.98101	.99634	.99941	.99992
.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO PF

.36788	.36788	.18394	.06131	.01533	.00307	.00051
.00007	.00001	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP

1.00000	1.00000	1.00000	.00000	.00000	.00000	.00000
1.00000	1.00000	1.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

PART NO. 3 *

SA 1 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 2 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 3 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 4 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 5 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP PF

1.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/AWP/VAR TO MEAN RATIO/TOTMU

.00000	.00000	.00000	2.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU PF

.13534	.27067	.27067	.18045	.09022	.03609	.01203
.00344	.00086	.00019	.00004	.00001	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU CDF

.13534	.40601	.67668	.85712	.94735	.98344	.99547
.99890	.99976	.99995	.99999	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO CDF

.40601	.85712	.98344	.99890	.99995	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO PF

.40601	.45112	.12631	.01547	.00105	.00005	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP

.54416	.75458	.72114	.00000	.00000	.00000	.00000
2.00000	2.00000	1.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

PART NO. 4 *

SA 1 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 2 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 3 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 4 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 5 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP PF

1.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/AWP/VAR TO MEAN RATIO/TOTMU

.00000	.00000	.00000	1.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU PF

.36788	.36788	.18394	.06131	.01533	.00307	.00051
.00007	.00001	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU CDF

.36788	.73576	.91970	.98101	.99634	.99941	.99992
.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO CDF

.36788	.91970	.99634	.99992	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO PF

.36788	.55182	.07664	.00358	.00008	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP

.37902	.71617	.52924	.00000	.00000	.00000	.00000
1.00000	1.00000	1.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

Exact Results for Sample Problem

INPUT DATA

PART	TYPE	#SA	QPHA	MU	SL	HA#	PUP
0	5.0	1.0	1.0	.0	.0	.0	.0
1	4.0	3.0	1.0	1.0	2.0	.0	1.0
2	3.0	.0	1.0	1.0	.0	1.0	1.0
3	3.0	.0	2.0	2.0	1.0	1.0	2.0
4	3.0	.0	2.0	1.0	.0	1.0	3.0

EXACT RESULTS

PART NO. 0 *

SA 1 BO CDF

.55499	.80204	.92915	.97872	.99446	.99872	.99973
.99995	.99999	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 2 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 3 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 4 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 5 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP CDF

.55499	.80204	.92915	.97872	.99446	.99872	.99973

.99995	.99999	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP PF

.55499	.24705	.12711	.04958	.01574	.00426	.00101
.00022	.00004	.00001	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

AMU PF

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU PF

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU CDF

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

BO CDF

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

BO PF

.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP

1.05943	$\boxed{.74224}$	1.42734	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

PART NO. 1 *

SA 1 BO CDF

.36788	.73576	.91970	.98101	.99634	.99941	.99992
.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 2 BO CDF

.40601	.85712	.98344	.99890	.99995	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 3 BO CDF

.36788	.91970	.99634	.99992	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 4 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 5 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP CDF

.05495	.58000	.90116	.97985	.99629	.99940	.99992
.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP PF

.05495	.52505	.32116	.07870	.01644	.00311	.00051
.00007	.00001	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

AMU PF

.36788	.36788	.18394	.06131	.01533	.00307	.00051
.00007	.00001	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU PF

.02021	.21337	.32141	.24705	.12711	.04958	.01574
.00426	.00101	.00022	.00004	.00001	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU CDF

.02021	.23358	.55499	.80204	.92915	.97872	.99446

.99872	.99973	.99995	.99999	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO CDF

.55499	.80204	.92915	.97872	.99446	.99872	.99973
.99995	.99999	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO PF

.55499	.24705	.12711	.04958	.01574	.00426	.00101
.00022	.00004	.00001	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP

1.05943	.74224	1.42734	.00000	.00000	.00000	.00000
1.66601	2.48845	.66950	.00000	.00000	.00000	.00000
.66601	1.48845	.44745	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

PART NO. 2 *

SA 1 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 2 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 3 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 4 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 5 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP PF

1.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

AMU PF

.36788	.36788	.18394	.06131	.01533	.00307	.00051
.00007	.00001	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU PF

.36788	.36788	.18394	.06131	.01533	.00307	.00051
.00007	.00001	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU CDF

.36788	.73576	.91970	.98101	.99634	.99941	.99992
.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO CDF

.36788	.73576	.91970	.98101	.99634	.99941	.99992
.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO PF

.36788	.36788	.18394	.06131	.01533	.00307	.00051
.00007	.00001	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP

1.00000	1.00000	1.00000	.00000	.00000	.00000	.00000
1.00000	1.00000	1.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

PART NO. 3 *

SA 1 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 2 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 3 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 4 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 5 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP PF

1.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

AMU PF

.13534	.27067	.27067	.18045	.09022	.03609	.01203
.00344	.00086	.00019	.00004	.00001	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU PF

.13534	.27067	.27067	.18045	.09022	.03609	.01203
.00344	.00086	.00019	.00004	.00001	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU CDF

.13534	.40601	.67668	.85712	.94735	.98344	.99547
.99890	.99976	.99995	.99999	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO CDF

.40601	.85712	.98344	.99890	.99995	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO PF

.40601	.45112	.12631	.01547	.00105	.00005	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP

.54416	.75458	.72114	.00000	.00000	.00000	.00000
2.00000	2.00000	1.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

PART NO. 4 *

SA 1 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 2 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 3 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 4 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

SA 5 BO CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP CDF

1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

AWP PF

1.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

AMU PF

.36788	.36788	.18394	.06131	.01533	.00307	.00051
.00007	.00001	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU PF

.36788	.36788	.18394	.06131	.01533	.00307	.00051
.00007	.00001	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

TOTMU CDF

.36788	.73576	.91970	.98101	.99634	.99941	.99992
.99999	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO CDF

.36788	.91970	.99634	.99992	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000
1.00000	1.00000	1.00000	1.00000	1.00000	1.00000	1.00000

BO PF

.36788	.55182	.07664	.00358	.00008	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

VAR/MEAN/VAR TO MEAN FOR BO,TOTMU,& AWP

.37902	.71617	.52924	.00000	.00000	.00000	.00000
1.00000	1.00000	1.00000	.00000	.00000	.00000	.00000
.00000	.00000	.00000	.00000	.00000	.00000	.00000

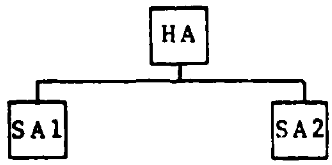
.00000	.00000	.00000	.00000	.00000	.00000	.00000

Appendix E

Type 1 Error Tests

Cases with No Type 1 Error
(HA stock=0 and HA QPHA=1)

The first five cases all refer to the hierarchy below:



Case 1:

PART DATA

Part	MU	Stock	QPHA
HA	1	0	1
SAs	1	0	1

HA BO RESULTS

	Mean	Variance
EXACT	2.524	2.034
APPROX	2.524	2.524
% error	0	24.1

Case 2:

PART DATA

Part	MU	Stock	QPHA
HA	1	0	1
SAs	1	0	2

HA BO RESULTS

	Mean	Variance
EXACT	2.026	1.312
APPROX	2.026	2.026
% error	0	54.4

Case 3:

PART DATA			
Part	MU	Stock	QPHA
HA	1	0	1
SAs	1	1	2

HA BO RESULTS		
	Mean	Variance
EXACT	1.497	1.330
APPROX	1.497	1.497
% error	0	12.6

Case 4:

PART DATA			
Part	MU	Stock	QPHA
HA	2	0	1
SAs	3	2	2

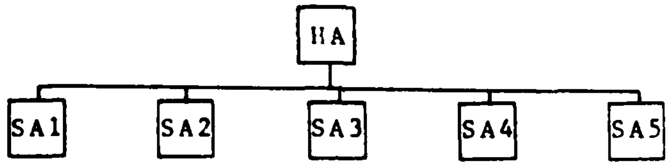
HA BO RESULTS		
	Mean	Variance
EXACT	3.230	2.701
APPROX	3.230	3.230
% error	0	19.6

Case 5:

PART DATA			
Part	MU	Stock	QPHA
HA	4	0	1
SA 1	1	0	1
SA 2	7	0	1

HA BO RESULTS		
	Mean	Variance
EXACT	10.905	10.377
APPROX	10.904	10.444
% error	≈ 0	.6

Cases six through nine use this hierarchy:



Case 6:

PART DATA

Part	MU	Stock	QPHA
HA	1	0	1
SAs	1	1	2

HA BO RESULTS

	Mean	Variance
EXACT	1.879	1.301
APPROX	1.879	1.879
% error	0	44.4

Case 7:

PART DATA

Part	MU	Stock	QPHA
HA	3	0	1
SAs	1	2	3

HA BO RESULTS

	Mean	Variance
EXACT	3.345	3.232
APPROX	3.345	3.345
% error	0	3.5

Case 8:

PART DATA

Part	MU	Stock	QPHA
HA	2	0	1
SAs	2	0	1

HA BO RESULTS

	Mean	Variance
EXACT	5.731	3.513
APPROX	5.731	5.731
% error	0	63.1

Case 9:

PART DATA

Part	MU	Stock	QPHA
HA	2	0	1
SA 1	1	0	1
SA 2	2	0	1
SA 3	3	0	1
SA 4	4	0	1
SA 5	5	0	1

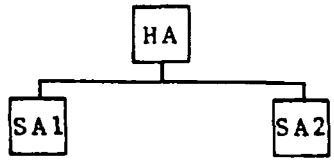
HA BO RESULTS

	Mean	Variance
EXACT	7.976	5.368
APPROX	7.976	7.976
% error	≈0	48.6

The mean errors that are not exactly 0 are caused by the probability distributions being truncated in the computer programs.

Cases with Type 1 Error
(HA stock not equal to 0 or HA QPHA not equal to 1)

Cases 10 through 13 use the following hierarchy:



Case 10:

PART DATA

Part	MU	Stock	QPHA
HA	1	2	1
SAs	1	1	2

HA BO RESULTS

	Mean	Variance
EXACT	.2498	.3696
APPROX	.2798	.4419
% error	12.0	19.6

Case 11:

PART DATA

Part	MU	Stock	QPHA
HA	1	0	3
SAs	1	0	2

HA BO RESULTS

	Mean	Variance
EXACT	1.054	.1533
APPROX	1.021	.2939
% error	-3.1	91.7

Case 12:

PART DATA

Part	MU	Stock	QPHA
HA	1	2	1
SAs	1	0	2

HA BO RESULTS

	Mean	Variance
EXACT	.4370	.6109
APPROX	.5571	.9223
% error	27.5	51.0

Case 13:

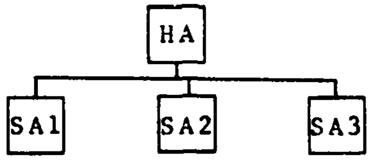
PART DATA

Part	MU	Stock	QPHA
HA	3	5	1
SA 1	1	0	1
SA 2	5	0	1

HA BO RESULTS

	Mean	Variance
EXACT	3.175	6.655
APPROX	3.188	6.748
% error	.4	1.4

Cases 14 and 15 use this hierarchy:



Case 14:

PART DATA

Part	MU	Stock	QPHA
HA	1	2	1
SAs	1	1	2

HA BO RESULTS

	Mean	Variance
EXACT	.2998	.4364
APPROX	.3556	.5722
% error	18.6	31.1

Case 15:

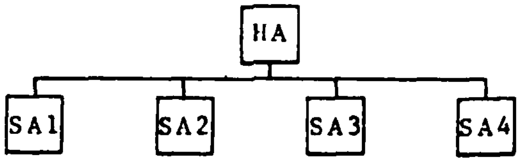
PART DATA

Part	MU	Stock	QPHA
HA	1	2	1
SA 1	1	0	1
SA 2	2	1	2
SA 3	1	0	2

HA BO RESULTS

	Mean	Variance
EXACT	.7422	1.059
APPROX	.8612	1.447
% error	16.0	36.6

Case 16:



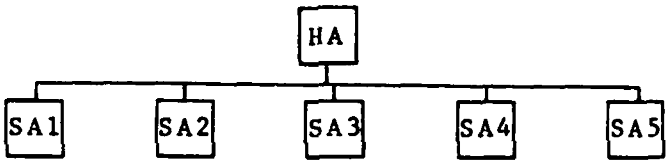
PART DATA

Part	MU	Stock	QPHA
HA	1	2	1
SAs	1	1	2

HA BO RESULTS

	Mean	Variance
EXACT	.3395	.4870
APPROX	.4191	.6823
% error	23.4	40.1

Case 17:



PART DATA

Part	MU	Stock	QPHA
HA	1	2	1
SAs	1	1	2

HA BO RESULTS

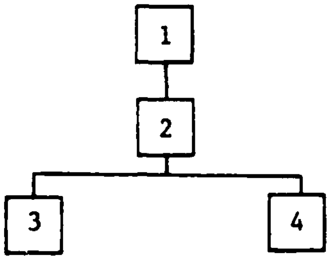
	Mean	Variance
EXACT	.3717	.5264
APPROX	.4714	.7732
% error	26.8	46.9

Appendix F

Type 2 Error Tests

Cases with No Type 2 Error
(Only 1 SA per HA above bottom level & no type 1 error)

Case 1:



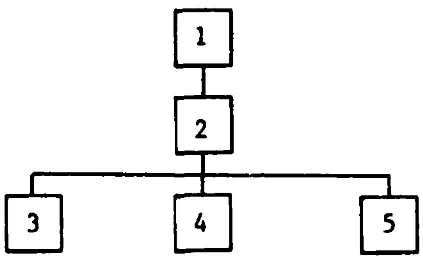
PART DATA

Part	MU	Stock	QPHA
1	4	0	1
2	1	0	1
3	3	2	4
4	2	3	2

PART 1 BO RESULTS

	Mean	Variance
EXACT	5.688	5.319
APPROX	5.688	5.688
% error	0	6.9

Case 2:



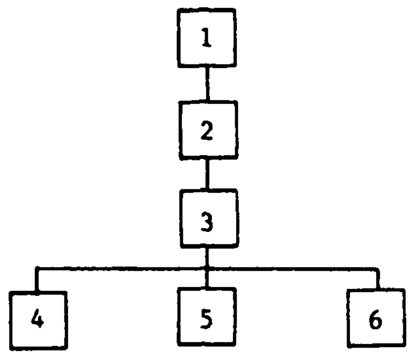
PART DATA

Part	MU	Stock	QPHA
1	1	0	1
2	2	0	1
3	1	0	1
4	2	0	1
5	1	0	1

PART 1 BO RESULTS

	Mean	Variance
EXACT	5.448	4.508
APPROX	5.448	5.448
% error	0	20.9

Case 3:



PART DATA

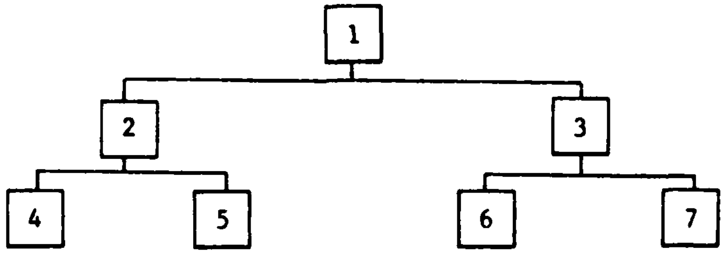
Part	MU	Stock	QPHA
1	1	0	1
2	3	0	1
3	1	0	1
4	2	3	2
5	1	0	3
6	3	2	4

PART 1 BO RESULTS

	Mean	Variance
EXACT	5.936	5.201
APPROX	5.936	5.936
% error	0	14.1

Cases with Type 2 Error

Cases four through six use the following hierarchy:



Case 4:

PART DATA			
Part	MU	Stock	QPHA
all	2	0	1

PART 1 BO RESULTS		
	Mean	Variance
EXACT	7.856	5.144
APPROX	7.985	7.927
% error	1.6	54.1

Case 5:

PART DATA			
Part	MU	Stock	QPHA
1	4	0	1
rest	2	0	1

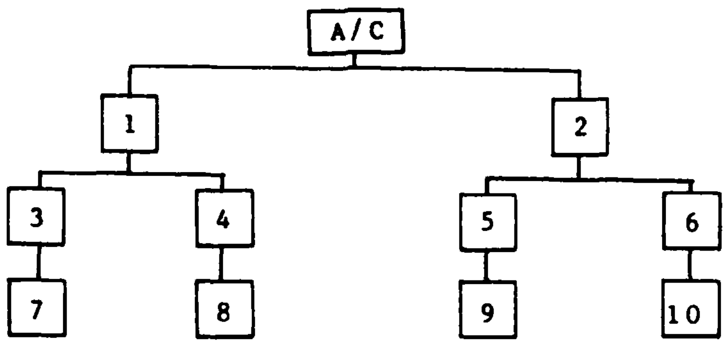
PART 1 BO RESULTS		
	Mean	Variance
EXACT	9.856	7.145
APPROX	9.987	9.987
% error	1.3	39.8

Case 6:

	PART DATA		
Part	MU	Stock	QPHA
1-3	.5	0	1
4-7	.5	1	1

PART 1 BO RESULTS		
	Mean	Variance
EXACT	1.634	1.347
APPROX	1.628	1.628
% error	-.4	20.9

The next case uses this hierarchy:



Case 7:

	PART DATA		
Part	MU	Stock	QPHA
1	2	0	1
2	1	0	1
3	3	0	1
4	1	0	1
5	1	0	1
6	2	0	1
7	1	0	1
8	2	0	1
9	3	0	1
10	2	0	1

A/C BO RESULTS		
	Mean	Variance
EXACT	7.587	3.942
APPROX	7.782	5.138
% error	2.6	30.3

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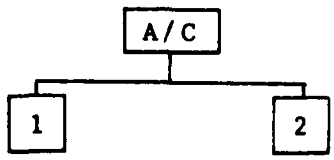
Appendix G

Sensitivity Analysis Results

Effect of Increasing Depth of Indenture

For the "single-level" hierarchy:



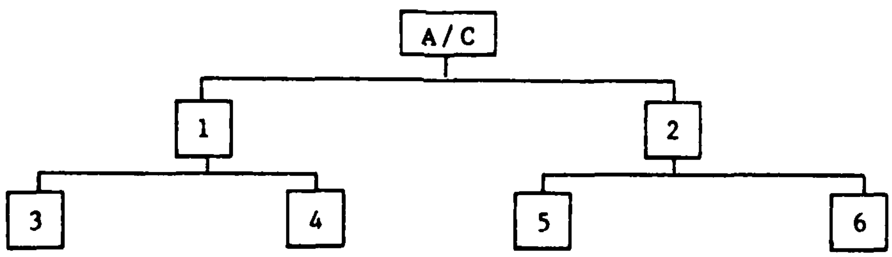
PART DATA

Part	MU	Stock	QPHA
1	2	3	2
2	1.5	1	2

A/C BO RESULTS

	Mean	Variance
EXACT	.6088	.4239
APPROX	.6088	.4239
% error	0	0

For the "two-level" hierarchy:



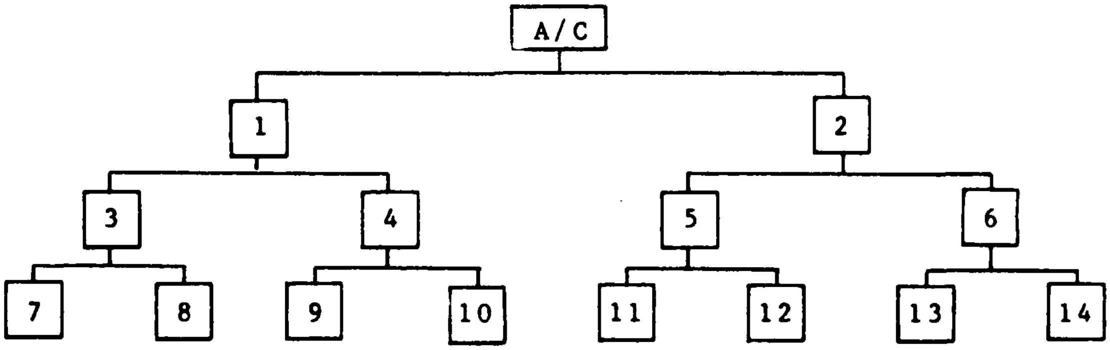
PART DATA

Part	MU	Stock	QPHA
odd #s	2	3	2
even #s	1.5	1	2

A/C BO RESULTS

	Mean	Variance
EXACT	.9422	.5245
APPROX	.9557	.5683
% error	1.4	8.4

For the "three-level" hierarchy:



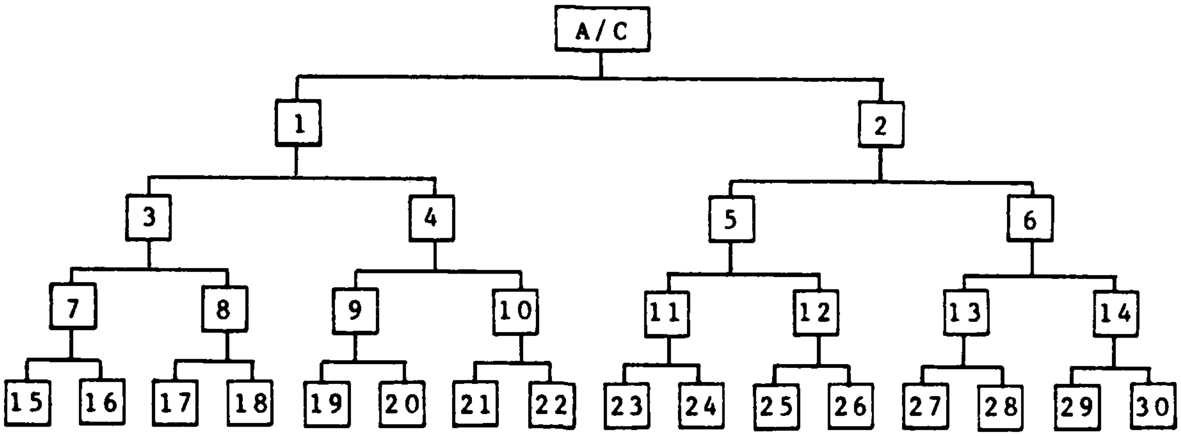
PART DATA

Part	MU	Stock	QPHA
odd #s	2	3	2
even #s	1.5	1	2

A/C BO RESULTS

	Mean	Variance
EXACT	1.117	.5410
APPROX	1.154	.6405
% error	3.3	18.4

For the "four-level" hierarchy:



PART DATA

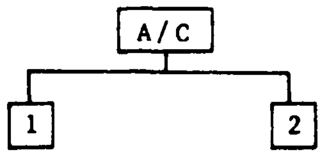
Part	MU	Stock	QPHA
odd #s	2	3	2
even #s	1.5	1	2

A/C BO RESULTS

	Mean	Variance
EXACT	1.206	.5418
APPROX	1.267	.6804
% error	5.0	25.6

Repeating the same sequence for a different data set gives the following results:

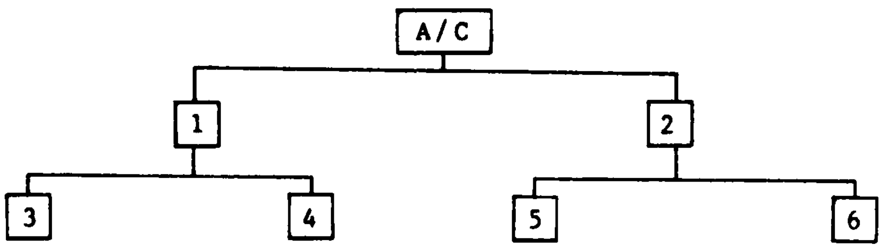
For the "single-level" hierarchy:



	PART DATA		
Part	MU	Stock	QPHA
all	2	3	1

	A/C BO RESULTS	
	Mean	Variance
EXACT	.4126	.6568
APPROX	.4126	.6568
% error	0	0

For the "two-level" hierarchy:

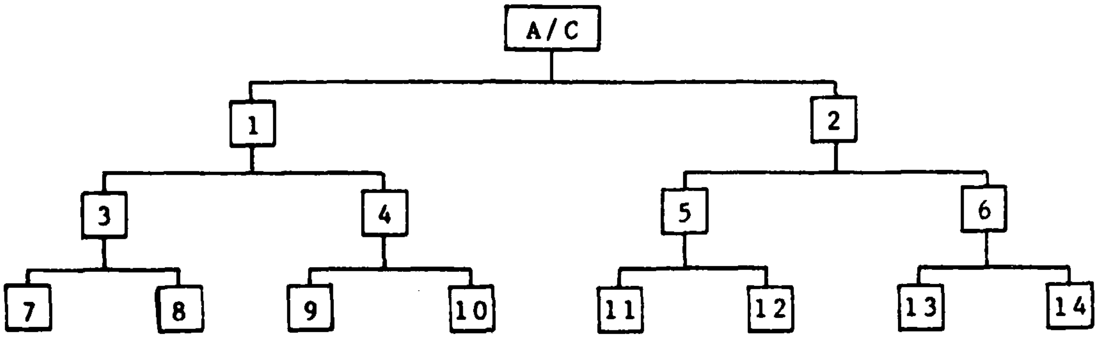


	PART DATA		
Part	MU	Stock	QPHA
all	2	3	1

A/C BO RESULTS

	Mean	Variance
EXACT	.7439	1.290
APPROX	.6875	1.093
% error	-7.6	-15.3

For the "three-level" hierarchy:



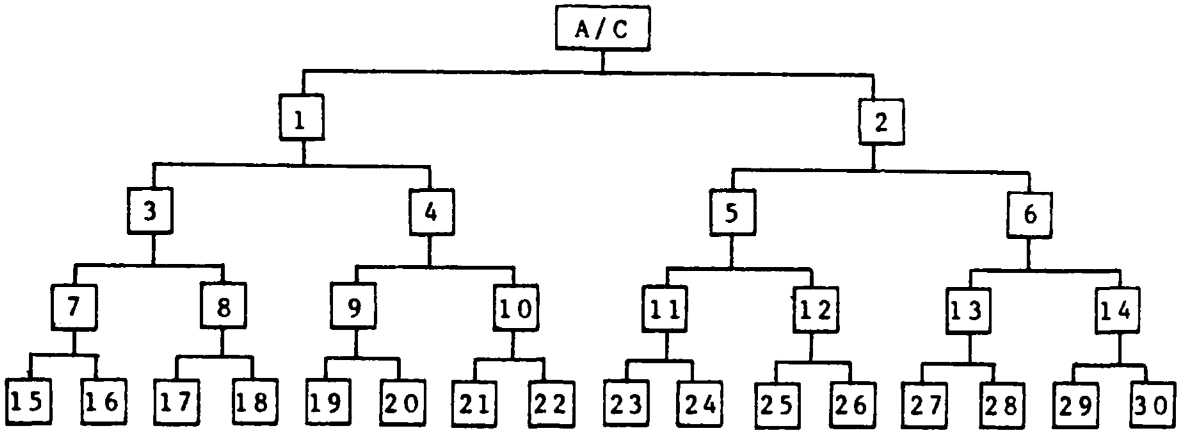
PART DATA

Part	MU	Stock	QPHA
all	2	3	1

A/C BO RESULTS

	Mean	Variance
EXACT	1.066	1.948
APPROX	.9043	1.419
% error	-15.2	-27.2

For the "four-level" hierarchy:



PART DATA

Part	MU	Stock	QPHA
all	2	3	1

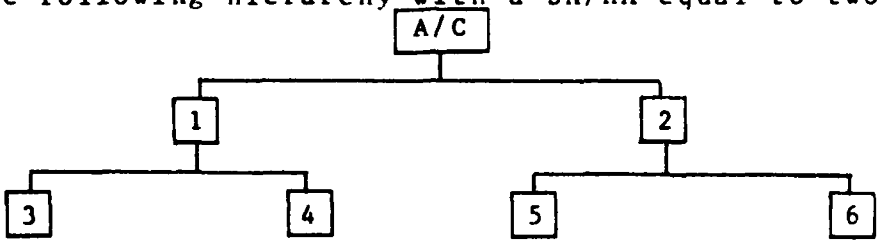
A/C BO RESULTS

	Mean	Variance
EXACT	1.405	2.643
APPROX	1.091	1.685
% error	-22.3	-36.2

Effect of the SA to HA Ratio

SET 1

For the following hierarchy with a SA/HA equal to two:



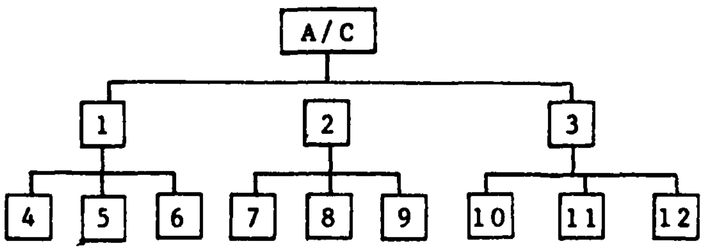
PART DATA

Part	MU	Stock	QPHA
all	1.5	2	2

A/C BO RESULTS

	Mean	Variance
EXACT	.5809	.4235
APPROX	.5896	.4379
% error	1.5	3.4

For an SA/HA of three:



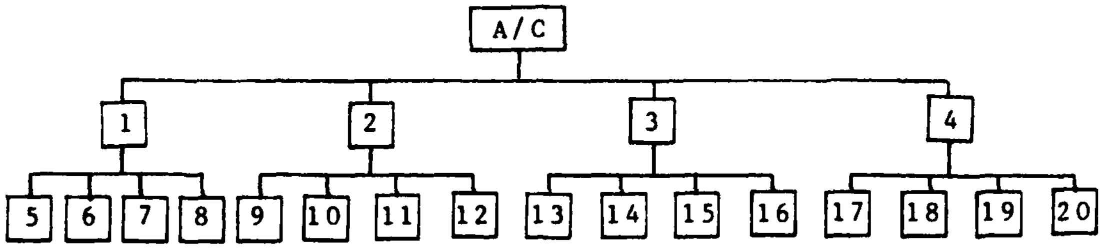
PART DATA

Part	MU	Stock	QPHA
all	1.5	2	2

A/C BO RESULTS

	Mean	Variance
EXACT	.8459	.4510
APPROX	.8728	.4879
% error	3.2	8.2

For an SA/HA of four:



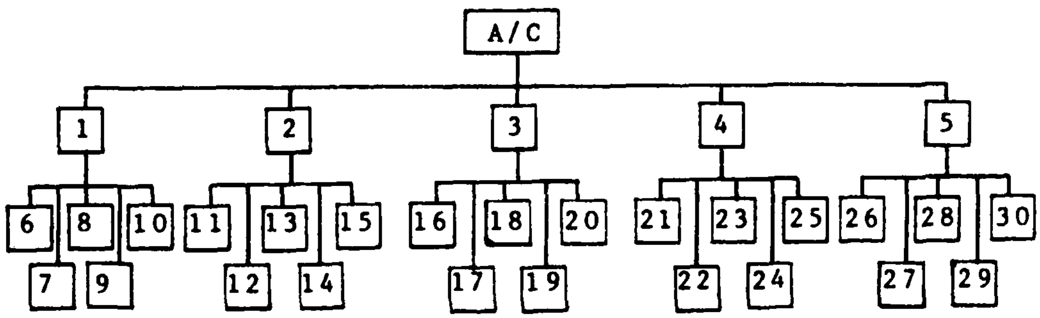
PART DATA

Part	MU	Stock	QPHA
all	1.5	2	2

A/C BO RESULTS

	Mean	Variance
EXACT	1.053	.4226
APPROX	1.106	.4822
% error	5.0	14.1

For an SA/HA of five:



PART DATA

Part	MU	Stock	QPHA
all	1.5	2	2

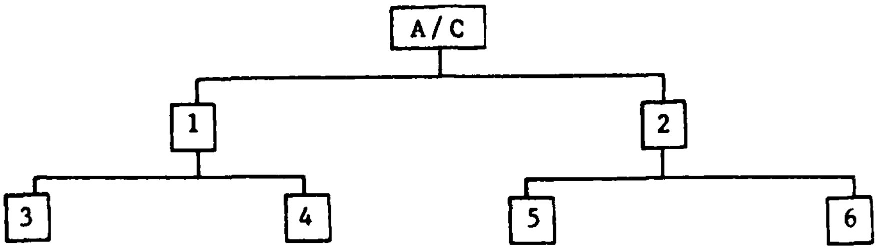
A/C BO RESULTS

	Mean	Variance
EXACT	1.210	.3912
APPROX	1.292	.4693
% error	6.8	20.0

SET 2

Repeating the same sequence with a different data set gives the following results:

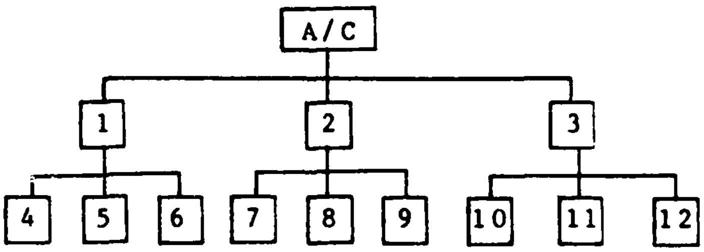
For the following hierarchy with a SA/HA equal to two:



	PART DATA		
Part	MU	Stock	QPHA
all	2	3	1

A/C BO RESULTS	Mean	Variance
EXACT	.7439	1.290
APPROX	.6875	1.093
% error	-7.6	-15.3

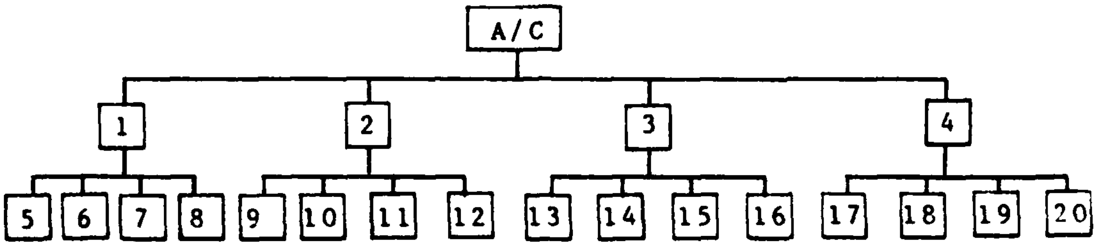
For an SA/HA of three:



	PART DATA		
Part	MU	Stock	QPHA
all	2	3	1

A/C BO RESULTS	Mean	Variance
EXACT	1.208	1.783
APPROX	1.125	1.530
% error	-6.9	-14.2

For an SA/HA of four:



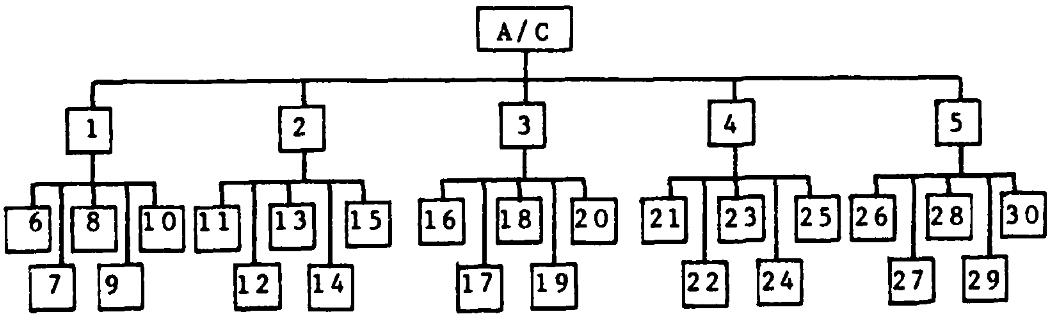
PART DATA

Part	MU	Stock	QPHA
all	2	3	1

A/C BO RESULTS

	Mean	Variance
EXACT	1.654	2.036
APPROX	1.562	1.794
% error	-5.6	-11.9

For an SA/HA of five:



PART DATA

Part	MU	Stock	QPHA
all	2	3	1

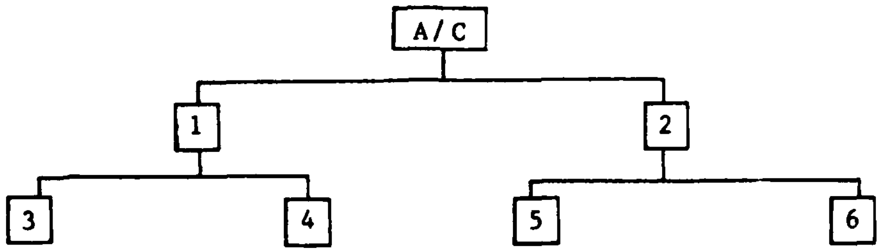
A/C BO RESULTS

	Mean	Variance
EXACT	1.876	2.056
APPROX	1.804	1.877
% error	-3.8	-8.7

SET 3

Repeating the same sequence with a different data set gives the following results:

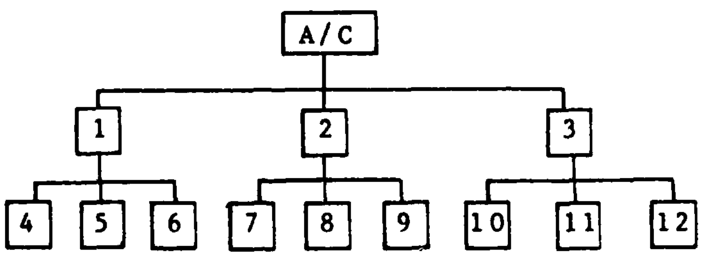
For the following hierarchy with a SA/HA equal to two:



	PART DATA		
Part	MU	Stock	QPHA
all	1	0	1

A/C BO RESULTS	Mean	Variance
EXACT	3.304	1.772
APPROX	3.397	2.222
% error	2.8	25.4

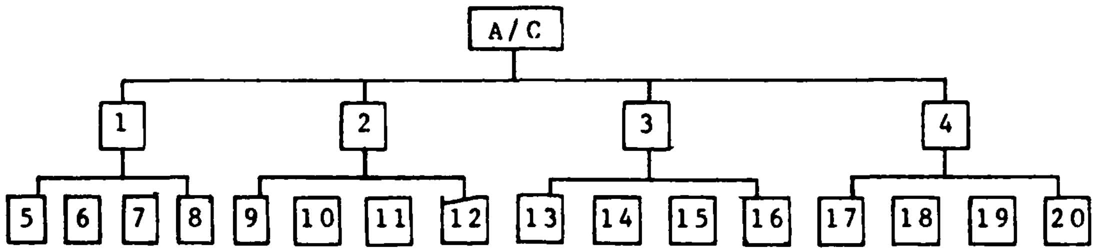
For an SA/HA of three:



	PART DATA		
Part	MU	Stock	QPHA
all	1	0	1

A/C BO RESULTS	Mean	Variance
EXACT	4.037	1.579
APPROX	4.286	2.241
% error	6.2	41.9

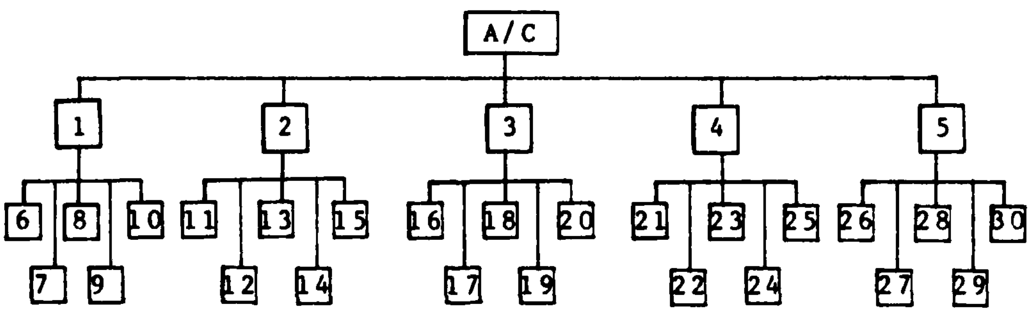
For an SA/HA of four:



Part	PART DATA MU	Stock	QPHA
all	1	0	1

A/C BO RESULTS	Mean	Variance
EXACT	4.533	1.458
APPROX	4.924	2.228
% error	8.6	52.8

For an SA/HA of five:



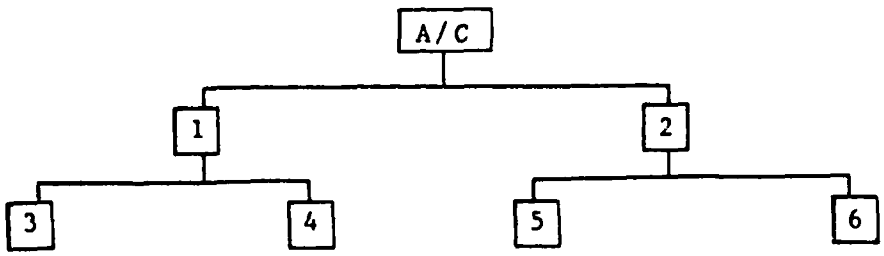
Part	PART DATA MU	Stock	QPHA
all	1	0	1

A/C BO RESULTS	Mean	Variance
EXACT	7.231	2.743
APPROX	7.992	5.229
% error	10.5	90.6

SET 4

Repeating the same sequence with a different data set gives the following results:

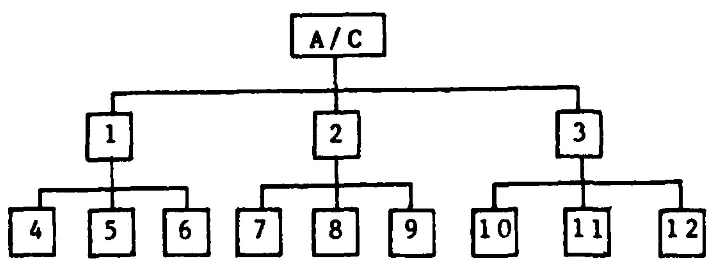
For the following hierarchy with a SA/HA equal to two:



Part	PART DATA MU	Stock	QPHA
all	1	3	1

A/C BO RESULTS	Mean	Variance
EXACT	.0579	.0841
APPROX	.0540	.0763
% error	-6.7	-9.3

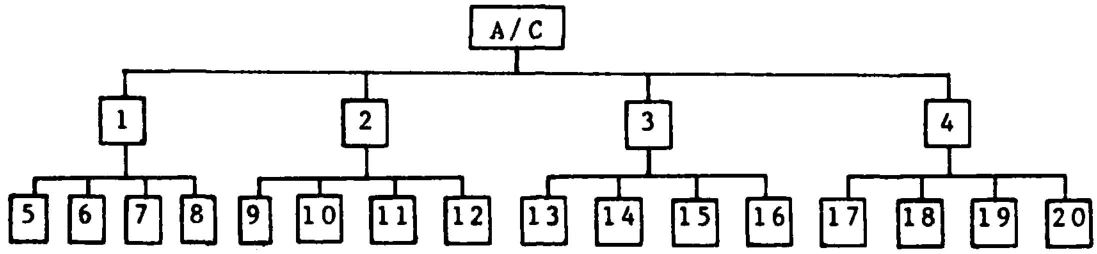
For an SA/HA of three:



Part	PART DATA MU	Stock	QPHA
all	1	3	1

A/C BO RESULTS	Mean	Variance
EXACT	.0944	.1366
APPROX	.0862	.1204
% error	-8.7	-11.9

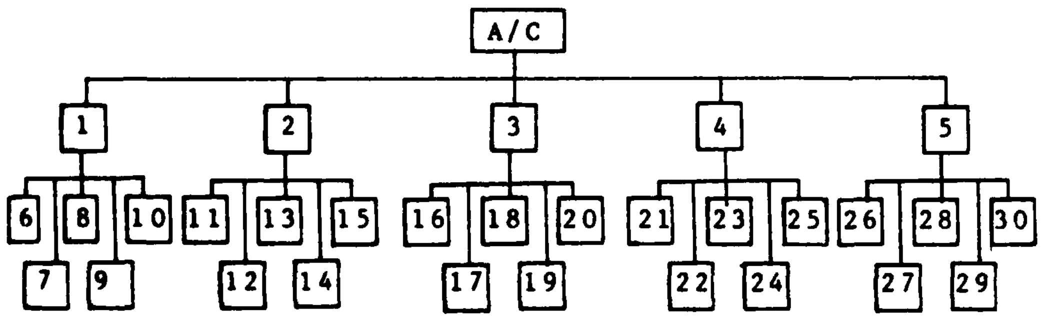
For an SA/HA of four:



Part	PART DATA MU	Stock	QPHA
all	1	3	1

A/C BO RESULTS	Mean	Variance
EXACT	.1354	.1940
APPROX	.1219	.1678
% error	-10.0	-13.5

For an SA/HA of five:

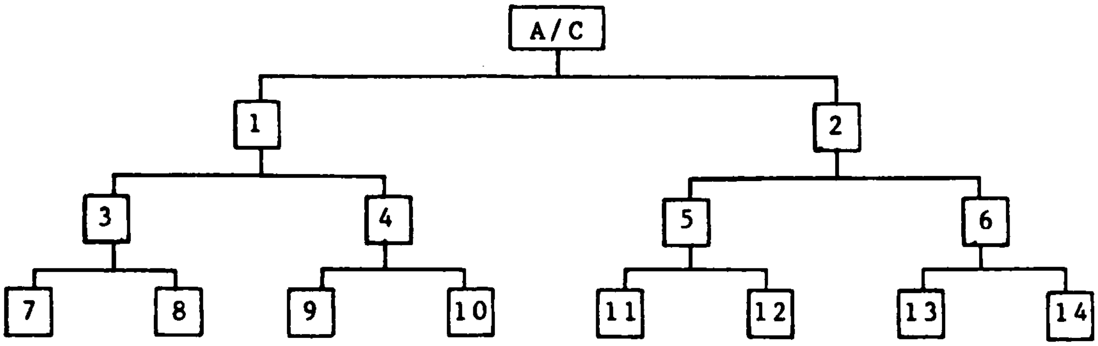


Part	PART DATA MU	Stock	QPHA
all	1	3	1

A/C BO RESULTS	Mean	Variance
EXACT	.0632	.0940
APPROX	.0556	.0787
% error	-12.0	-16.3

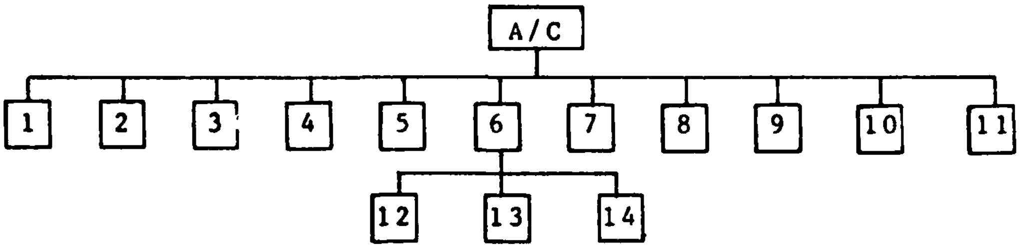
Effect of Stock Level and QPHA

This hierarchy with all MUs equal to two was used to generate the results in the following table:



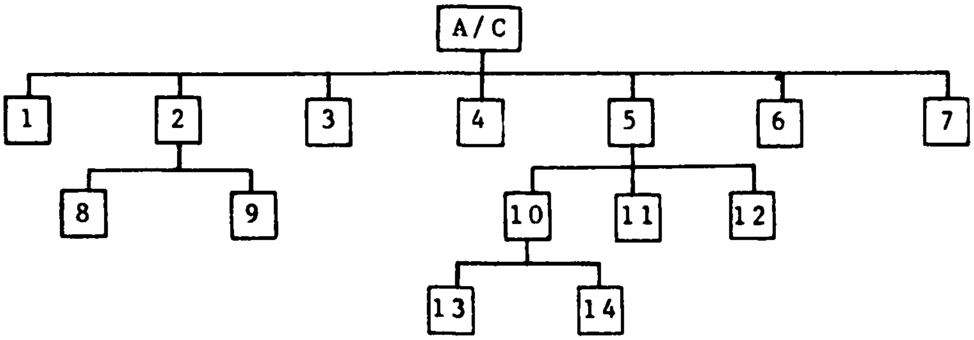
Stock Level		QPHA 1 mean	QPHA 1 var.	QPHA 2 mean	QPHA 2 var.	QPHA 3 mean	QPHA 3 var.
0	APPROX	9.569	6.289	3.145	1.099	1.941	.4306
	EXACT	9.119	4.131	2.946	.6146	1.848	.3105
	% error	4.9	52.2	6.8	78.8	5.0	38.7
1	APPROX	6.222	4.733	2.195	.8614	1.433	.3792
	EXACT	6.122	4.122	2.071	.6156	1.353	.2840
	% error	1.6	14.8	6.0	40.0	5.9	33.5
2	APPROX	3.061	3.234	1.248	.7075	.9184	.3542
	EXACT	3.215	3.825	1.201	.6101	.8900	.3027
	% error	-4.8	-15.5	3.9	15.9	3.2	17.0
3	APPROX	.9043	1.420	.4826	.4271	.4061	.2860
	EXACT	1.066	1.948	.4766	.4183	.3952	.2772
	% error	-15.2	-27.1	1.3	2.1	2.8	3.2
4	APPROX	.2060	.3516	.1406	.1496	.1282	.1182
	EXACT	.2465	.4648	.1418	.1511	.1278	.1173
	% error	-16.4	-24.4	-.9	-1.0	.3	.8
5	APPROX	.0500	.0829	.0382	.0421	.0360	.0358
	EXACT	.0548	.0943	.0385	.0425	.0360	.0358
	% error	-8.7	-12.1	-.7	-.8	.06	.03

This hierarchy with all MUs equal to 2.7 was used to generate the results in the following table:



Stock Level		QPHA 1 mean	QPHA 1 var.	QPHA 2 mean	QPHA 2 var.	QPHA 3 mean	QPHA 3 var.
0	APPROX	7.185	5.137	3.193	.8176	2.191	.3253
	EXACT	7.085	3.906	3.124	.6357	2.152	.2675
	% error	1.4	31.5	2.2	28.6	1.8	21.6
1	APPROX	5.420	3.872	2.551	.7039	1.795	.3507
	EXACT	5.355	3.361	2.499	.5934	1.765	.3195
	% error	1.2	15.2	2.1	18.6	1.7	9.8
2	APPROX	3.779	2.842	1.926	.6268	1.402	.3076
	EXACT	3.774	2.812	1.859	.5735	1.378	.2869
	% error	.1	1.1	3.6	9.3	1.7	7.2
3	APPROX	2.333	2.168	1.328	.5633	1.052	.2678
	EXACT	2.369	2.319	1.317	.5457	1.043	.2574
	% error	-1.5	-6.5	.8	3.2	.9	4.0
4	APPROX	1.197	1.539	.7704	.5108	.6631	.3191
	EXACT	1.231	1.650	.7700	.5088	.6602	.3172
	% error	-2.8	-6.7	.1	.4	.4	.6

This hierarchy with all MUs equal to 1.5 was used to generate the results in the following table:



Stock Level		QPHA					
		1		2		3	
		mean	var.	mean	var.	mean	var.
0	APPROX	6.031	4.038	2.453	.6057	1.666	.3209
	EXACT	5.845	3.076	2.326	.4140	1.586	.2813
	% error	3.2	31.3	5.5	46.3	5.0	14.1
1	APPROX	3.551	2.426	1.684	.4912	1.233	.2127
	EXACT	3.533	2.342	1.615	.4109	1.183	.1694
	% error	.5	3.6	4.3	19.5	4.2	25.6
2	APPROX	1.649	1.445	.9875	.4065	.8449	.2279
	EXACT	1.711	1.625	.9720	.3873	.8340	.2192
	% error	-3.6	-11.1	1.6	5.0	1.3	4.0
3	APPROX	.5554	.7102	.4024	.3110	.3735	.2482
	EXACT	.5860	.7811	.4016	.3102	.3711	.2470
	% error	-5.2	-9.1	.2	.3	.6	.5
4	APPROX	.1475	.2143	.1169	.1157	.1117	.1014
	EXACT	.1525	.2243	.1171	.1159	.1116	.1013
	% error	-3.3	-4.5	-.2	-.2	.1	.1

VITAE

Steven Wilmar Weiss was born on 26 July 1952 in Frankenmuth, Michigan. He graduated from Frankenmuth High School in 1970 and subsequently attended the United States Air Force Academy from which he received a Bachelor of Science degree in the field of Mathematics. Upon graduation, he accepted a commission in the USAF and entered pilot training. In July 1975 he received his wings and was assigned to the 7th Airborne Command and Control Squadron, Keesler AFB, Mississippi. He served there as an EC-130 pilot and flight instructor until entering the School of Engineering, Air Force Institute of Technology, in August 1981.

Permanent address: 7905 Junction Rd.
Frankenmuth, Michigan 48734

Neil Arthur Youngman was born on 1 February 1948 in Massillon, Ohio. He graduated from Fairless High School and attended The Ohio State University from which he received a Bachelor of Science degree in Mathematics. After graduation, he completed Officer Training School and received a commission in the United States Air Force in March 1972. Upon completion of the U.S. Army Helicopter School he was assigned to the Aerospace Rescue and Recovery Service, serving tours in Florida and Okinawa. Following the Fixed Wing Transition Course, he served as a C-130 pilot and flight examiner at McChord AFB, Washington until entering the Graduate Program in Strategic and Tactical Sciences at the Air Force Institute of Technology, in August 1981.

Permanent address: 12374 Lawndell Rd., S.W.
Beach City, Ohio 44608

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9 SEP 1983

19. KEY WORDS (Continue on reverse side if necessary and identify by block number)

WARS	Poisson	Wartime-Requirements Assessment
WRSK	Spares	
BLSS	Readiness	
WRM	Materiel	

20. ABSTRACT (Continue on reverse side if necessary and identify by block number)

The purpose of this study was to determine the relative effect of using the Poisson approximation in the Wartime Assessment and Requirements System (WARS) assessment subroutine and to gain a better understanding of the exact error-causing mechanism involved.

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(Block 20 continued)

To accomplish this two Fortran computer programs were developed, one to compute the expected number of not mission capable aircraft using the accurate mathematical calculations, the other to compute the same figure using the Poisson approximation. These programs were used to evaluate the approximation caused error for different parts hierarchies and data sets.

The analysis identified two distinct causes for the error induced by the approximation. These sources of error were confirmed by running test cases specifically tailored to eliminate the error-causing characteristics and noting that no approximation error resulted.

The approximation error was found to fluctuate in sign and magnitude for different cases. Sensitivity analysis was performed to identify the sensitive parameters.